BRITISH ASSOCIATION

EETING IN SOUTH AFRICA 1

THE TEACHING OF
LEMENTARY MECHANIC

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BRITISH ASSOCIATION MEETING

In South Africa, 1905

DISCUSSION AT JOHANNESBURG ON

THE TEACHING OF ELEMENTARY MECHANICS

Which took place on August 29th, 1905, in Section A

Professor FORSYTH

President of the Section, in the Chair

Edited by JOHN PERRY

To which is added a paper by C. E. Ashford, M.A., on "The Teaching of Mechanics by Experiment," read at the York Meeting of The British Association, 1906

London
MACMILLAN AND CO., Limited
NEW YORK: THE MACMILLAN COMPANY
1906

Richard Clay and Sons, Limited,
BREAD STREET HILL, E.C., AND
BUNGAY, SUFFOLK

PROCEEDINGS

UNTIL the morning of the day on which the discussion took place I thought that there was to be a Joint Meeting of Sections A and G. It is a pity that there was any misunderstanding, because the views on the teaching of mechanics of the engineers attending Section G would have been of very great value. I secured a verbatim report, and sent a copy of his remarks to each speaker for correction. It was understood that if a speaker did not make corrections (alterations being in square brackets) they would be left to myself and the printers. After my return to England I sent a great number of copies of my address to men whose opinions would have weight, but I received only eleven replies. I promised at Johannesburg to publish a reply dwelling especially on my reason for recommending the use of engineers' units in schools and colleges. On this subject I venture to direct the attention of readers to the letters which appeared in *Nature*, Vol. lv. (1896–7), pages 124, 176, 223, 247, 270, 293, 317, 365, 366, 389, 415, 439, and to the review of a book, page 49. The Editor of *The School World* and Mr. Ashford have given permission for the publication of Mr. Ashford's Paper.

JOHN PERRY.

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ON THE TEACHING

OF

ELEMENTARY MECHANICS

By Professor John Perry.

I may disagree with other speakers in believing boys to be so uneducated at present that for one of them to learn mechanics is almost miraculous. I mean to speak only of those boys of the future who may be fit to study the subject. Again, I shall probably differ from other speakers in what is meant by "a knowledge of elementary mechanics." I do not consider the boy who passes so easily the ordinary examinations in mechanics, so quickly answers questions concerning Attwood's machine,[1] for example, to have the knowledge I speak of. He remembers a few formulæ, but he has no comprehension of the simplest mechanical principles, and when he forgets the formulæ, as he very soon does, he forgets everything, and dislikes the thought of mechanical theory.

The remarks I shall make are applicable to the training of all boys. It must not be assumed that I am prejudiced in favour of an engineering training for all men because I happen to illustrate my ideas through my experience in regard to engineers and others in whose professions applied physical science is important.

By physical science I mean the study of physics and chemistry and geology and mechanics, using experiment, observation, and mathematical or other reasoning. Its study involves much arithmetical, algebraic and other computation in dealing with the results of observation and experiment; it involves sketching and

[1] I may be misunderstood here; I believe in the use of a cheap Attwood's Machine, but many other contrivances will also, I hope, be invented and used, so that the fundamental idea may be given to a boy in many different ways. In all such apparatus I object to complications, refinements and adjustments introduced to satisfy the teacher's weak desire for perfection; they only confuse the average boy and make it nearly impossible for him to acquire the one simple idea which we wish to convey.

the description in English of the apparatus and the phenomena observed. It involves practice in the use of books and criticism of accounts of experiments made by other people. The applications of physical science involve not merely the use of mere knowledge, but the use of all one's experience, all the habits of thought produced by the study of physical science. There is no mental faculty which such a study has not helped to develop and does not continue to use and concentrate upon new problems. There is no finality possible in such study; a man knows himself to be a student always, he is always learning. How far are we at present from this ideal condition of things! We find that when average boys leave school they cannot compute, they cannot describe in English anything they have seen; they know almost nothing of elementary science.

The first year after entering a Technical College is wasted in the study of school subjects. If boys enter at once on an apprenticeship they have no foundation on which to build a scientific knowledge of their profession. It does not seem at first sight to be in consequence of this that the curriculums of Technical Colleges are becoming quite too complex, but it really does follow. Attempts are being made to teach everything that an engineer is likely to want in his profession. It results that a three years' course is found to be too short; a four years' course is found to be too short; and five years' courses are already suggested in America. These absurdities are due to the fact that students have been badly educated, they are not fond of reading; consequently they learn the science of their profession at the college only; we cannot depend upon their private study afterwards. It is becoming forgotten that the true function of a Technical College is to prepare a man for the continued study of the science of his profession all through his life; to make him anxious to study all such subjects as touch his profession in any way; to interest him even in the study of many subjects which hardly touch his profession at all, subjects interesting to him as a citizen, as an intelligent inhabitant of a very wonderful world.

Boys who do not enter a Technical College, but proceed from school directly to practical work, are, of course, in a worse case still.

The fairly clever young engineer is found to know something of

motion in one direction—motion of translation. He is very weak in the rotational motion of a rigid body. He does not know that *force* is (vector) rate of change of momentum. He may have had an extensive course in mechanics, but as he does not teach the subject he forgets all the labour-saving rules with which he worked difficult problems so readily. He ought to have simple principles well in hand, ready to use, however clumsily, on any problem; but he does not possess them.

The average man who once upon a time was crammed for examination by formulæ is found to know nothing of kinetics, although he may retain some of his knowledge of graphical statics.

I need not speak of a third kind of man, the very mathematical man; he often does not know anything of mechanics; it is the subject of applied mathematics that he has studied and that he cares for.

The evil is inherent in our whole system of so-called education. The Universities and Technical Schools use bad methods because the public schools fail to send up intelligent boys; the public schools are inefficient because the preparatory schools are inefficient, and the preparatory schools have a right to blame the earlier training of the boys that they receive.

Children should in their play be accustomed to measurement; playing at keeping shop, selling things to each other by weight and measurement, paying for things in actual money. Measurement of things with callipers and scales would accustom boys of eight to the use of decimals. Things requiring memory can be learnt only in early youth; weights and measures, the multiplication table, languages, plays involving spelling, the repetition of poetry, surely these are the things that ought to occupy children when very young. The reasoning powers will gradually develop in a healthy way if only they are not forced; powers of memory and of observation are at their best and only need guidance to equip the boy with stores of classified facts. Thus too, this early time is the time to let a child find out for himself that if he can read he can use story books and if the people about him are fond of reading he is certain to become fond of reading also.

I take it that it is not much before the age of ten that the average boy begins consciously to reason for himself, and after that time

he may sketch and draw plans of his schoolhouse and the roads or streets about it; he soon can use a map when walking or cycling, and he knows that maps may be of different scales. I would lead him up to the vector subject of geometry only slowly, through maps useful to himself, through problems on heights and distances and the like. Do you remember a story called "Sandford and Merton," where a Mr. Barlow taught two boys by methods surely the most interesting and delightful? It is between the years of nine and thirteen that boys need a Mr. Barlow, and I can imagine one Mr. Barlow being able to look after ten boys and perhaps more. There were no formal courses of study in many subjects such as exist in schools, each in its watertight compartment. One night the boys are interested in the stars; that night he tells them about the stars and lets them look through a telescope. He gives them stars and solar system just so long as they are interested. He uses a globe as well as mere maps in teaching them geography, but the soul-destroying idea of a course of study on "the use of the globes" does not commend itself to him. By pleasant experience they learn that there are rules about levers and parallel forces, and with simple apparatus he illustrates these rules. This is the time, say at the ages of eleven to thirteen, when boys ought to have a course of experimental science, weighing and measuring accurately, learning the rules of mensuration, taking specific gravities, learning something of barometers and thermometers, of magnetism and currents of electricity. People do not seem to know how cheaply and easily quite a lot of interesting apparatus may be rigged up by boys. They make glass-rubbed electric machines and Leyden jars out of bottles bought cheaply from grocers. They can arrange an electric telegraph, and after they have learnt the code, they can signal all sorts of messages to each other in all sorts of ways, not merely electrically, educating the senses of hearing and sight and touch. Their hands are educated by the use of tools, and so they learn all sorts of properties of materials. Thus I advocate a course of elementary science which, to be followed by a boy, involves the continual use of computation of all kinds; which keeps a boy interested; which satisfies his voracious curiosity; which is first qualitative, and may be made just as quantitative as the boy and teacher please to make it.

To the age of fourteen, although the boy has learnt elementary trigonometry and the use of maps, he may be said to have studied merely scalar quantities. He knows some algebra, he can use logarithms, he can use squared paper in all sorts of common-sense ways, he knows what is meant by speed, he can differentiate and integrate x^n, and has had many applications of the use of the calculus. He has a fairly clear notion of the various forms of energy and their equivalent values; he knows about the law of work; about friction; he has experimented on the efficiency of machines and the loss of energy by friction; the efficiency of all kinds of motors, especially of simple lifting machines, electric motors, &c., which can be experimented with easily.

As he has methods of calculating other forms of energy, so he can calculate concerning kinetic energy. A frictionless switch-back railway or the bob of a pendulum gives him an excellent illustration of a store of energy which remains constant in amount whilst changing its character continually. He now measures the strength and stiffness of wires and beams, and gets acquainted with the various simple stresses and strengths of materials.

It may be said that at the age of fourteen he has that experience and knowledge which will enable him to comprehend a vector subject, and he may now begin graphical and experimental work in the addition and subtraction of vectors such as displacements, velocities and forces. The easiest illustration of the usefulness of such work is through the use of strings and weights and pulleys. Forces in equilibrium that do not act through a point are quite easily dealt with, and the boy's knowledge of trigonometry enables the algebraic and graphical treatments of the subject to go on concurrently. I think it most essential that both methods should be familiar to the student from the beginning.

The two elementary principles of statics (that if forces are in equilibrium their vector sum is zero, and the sum of their moments about any axis whatsoever is zero—or, in other words, the vector polygon is closed and the link polygon is closed) ought to be so clear to a student, they should have been presented to him from so many points of view, that it is practically impossible

for him to forget them. They ought to be as much a part of his mental machinery as the power to walk is part of his physical function. The analogy between linear and angular displacements, forces and moments, work done through linear displacement and work done through angular displacement, is not difficult to comprehend if the student is experimenting with motors; torque multiplied by angular velocity being power; half inertia multiplied by v^2 and half moment of inertia multiplied by a^2 ought to be easily understood at this stage.

The subject of relative motion is always troublesome. Dealing as I have often done with young engineers, I have found the wheels of turbines or centrifugal pumps to give the easiest illustrations; how fluid may be directed to enter a wheel without shock; the velocity of the fluid relatively to the wheel and its velocity relative to the case become of great interest a little later.

The great law of mechanics that when force does not act on a system from the outside the total momentum in any direction remains constant may be made very clear by experiment. That rate of change of momentum is force; this, again, must be made clear by experiment, and again I think that experiments with fluids give the best illustrations.

Motion being in one direction, it is quite easy to show that definitions of work and storage of kinetic energy are in agreement with Newton's definition of force; and, whether by means of the rather intricate and misleading Attwood or by other more modern and simpler methods of experiment, much time ought to be spent upon this subject.

We come now to a subject which is exceedingly easy from the mere mathematician's point of view, but which seems to me exceedingly difficult, namely, that force is the rate of change of momentum; momentum being at last considered as a vector quantity. I need hardly say that the student usually has no knowledge of rate of change of a vector quantity. He has already considered force when there is no change of direction; let him now, before taking up the general case, consider force when there is change of direction but no change of speed. It seems to me that he must take up this subject either from astronomical illustrations and talk of centripetal force—a method

the most difficult for the average boy—or from the laboratory experiment point of view, and talk of centrifugal force—a method which I think to be quite easy, to present almost no difficulties. There is apparatus which measures with great exactness the centrifugal force exercised on its constraints by a body of any inertia, describing a circle of any radius at any speed. Then there is the conical pendulum, which by itself would be unsatisfactory, but which is wonderfully satisfactory when the idea has been already obtained otherwise.

The idea of centrifugal force, calculations involving computation of centrifugal force, may easily come into a student's work at quite an early period; it is a very different thing to reason about the matter. Centripetal force which continually acts, doing no work, yet causing continual change, is surely one of the most novel ideas to students, and familiarity cannot destroy our wonder.

I believe that the idea of force in general as vectorial rate of change of momentum can only be clearly seen through two kinds of illustration: the first the Newtonian kind, the astronomical kind; the second being of the hydraulic kind.

It will be seen throughout this statement of mine that I lay no stress upon mere abstract proofs of propositions in mechanics. To understand, to comprehend, these are difficult; these need numerous experiments and illustrations. When understanding is effected there is no difficulty about the proofs. It is quite usual to find men who can prove everything without having any comprehension of what they have proved.

Again, let us carefully consider what our aim is. If our aim is to enable men to pass examinations by outside examiners, even if this is only part of our aim, if the matter is carefully considered it will be found that however subtle the examination system may be, there can be no true teaching of mechanics. With a due sense of responsibility I say that an outside examination system and true education are antagonistic to one another.

Huxley prophesied what has since come to pass: that if the colleges of London became constituent colleges of one University, all true education in London would begin to disappear. How correct he was! Initiative and originality are gradually disappearing from courses of instruction. London University is a manu-

factory of B.Sc.'s, who resemble one another exactly as buttons do. As regards such a subject as mechanics, the examination system requires that a student should have labour-saving rules for every kind of problem, enabling him to give an answer in a very limited time. Such labour-saving rules can only be remembered by a man who is always teaching the subject; they are soon forgotten by an engineer, who blames himself and loathes the subject ever after. Besides, they destroy the power of thought.

When I was in Japan it was my custom to give a course of only three weeks' duration on graphical statics to my engineering students. A friend who taught at another institution had a class of much the same kind of student, and he laughed at the idea that such a short course as mine could be of any use. He, following the German system, devoted a whole year to this subject, showing his students how every kind of problem was worked. I gave a few general principles, and told my students that by the exercise of common sense any problem whatsoever could be solved by them—perhaps in a longer time than if they had labour-saving rules, but solved with certainty.

My critic and I agreed that he was to set a paper, to be given to both classes—to his, which had solved hundreds of problems, and to mine, which depended on a few principles thoroughly well known to them. I stipulated for plenty of time and also that his problems should be as new to his students as they certainly would be to mine.

Every one of my students worked the paper completely, and I sent him their answers. He never sent me the worked papers of his students. I venture to think that there is a valuable object lesson in this true story.

I will now pass round copies of the report of the M.A. Committee on the Teaching of Elementary Mechanics. With that report I may say that I generally agree, but I cannot admit that the difficult subject of kinetics can ever be taught before or even along with statics. The suggestion that merely easy numerical trigonometry should precede the theoretical study of mechanics shows that the Committee have very different views from mine; I would have the student know how to apply the principles of the infinitesimal calculus in his study of kinetics from the theoretical side. I have objections to some of the details of the report; for

example, the academic *poundal* ought never to be used; students ought to recognise only two sets of units, the C.G.S. and engineers' units. Also I would certainly not abandon the phrase "centrifugal force."

As there is no formal report from the Committee appointed by the British Association to investigate this subject, I should like to say that, as secretary, I sent a printed letter to each of the 612 secondary schools whose names are given in Whitaker's Almanac, asking the following questions concerning the teaching of mechanics at that school:—

1. At what stages does a boy take mechanics?

2. Does he take mechanics as a part of experimental physics, or as a part of mathematics, or how?

3. If he takes mechanics both experimentally and mathematically in the same term, is he under the same master for both methods?

4. At what age does the average boy clearly understand force to be (equal to) * rate of change of momentum?

In all 187 answers were received; in nine cases the whole answer was that no mechanics was taught at that school. Speaking, then, of the 178 remaining cases:—

Question 1 was badly framed, and the answers do not lend themselves to any kind of comparison.

Questions 2 and 3.—62 per cent. take mechanics both mathematically and as a part of experimental physics; 15 per cent. take mechanics only as a part of mathematics; 21 per cent. take mechanics as a part of experimental physics and only in this way; 31 per cent. say that the boys are under the same master for experimental and mathematical mechanics; 30 per cent. say that the boys are under different masters for experimental and mathematical mechanics.

Question 4.—Two answers "12 to 13," three answers "13 to 14," nineteen answers "14 to 15," eighty-six answers "over 15 years," and the answer "Never" is given in thirteen cases.

* I do not object to a man's saying "equal to" or to another man's saying "proportional to," but to students of the higher philosophy force *is* rate of change of momentum.

REPORT OF THE MATHEMATICAL ASSOCIATION COMMITTEE ON THE TEACHING OF ELEMENTARY MECHANICS.

THE Committee make the following suggestions:—

A. PRELIMINARY EXPERIMENTAL WORK.

1. That a simple course of experiments would be useful with a view to illustrating—
   (α) Composition and resolution of forces.
   (β) The principle of the lever.
   (γ) Friction.
2. The experiments should, if possible, aim at discovery, and be quantitative—mere verification is less useful.

B. GENERAL REMARKS ON EXAMPLES AND METHODS.

3. Examples should at first, as a rule, be numerical, and any of a general character should conclude with a numerical application.
4. Examples should, as far as possible, be of a practical nature.
5. A specially instructive class of example consists in compiling a table or drawing a graph to show the effect on a result of variation in a certain datum.
6. Stress should be laid on the great importance of checking results by an independent method; in particular, questions should often be worked out both graphically and by calculation.
7. Prominence should be given to geometrical methods.
8. Pupils should always be required to specify the body whose equilibrium or motion is being considered, and to indicate the complete system of forces acting on the body, before writing down any equations.

9. Simplifying assumptions, such as that friction, stiffness of ropes, weights of certain bodies, etc., are being disregarded in any particular question, cannot be too explicitly stated.
10. Fancy names for technical terms are to be avoided.

C. Statics.

11. As the basis of the subject the parallelogram of forces should be assumed as an experimental result.
12. This should be immediately followed by problems on the equilibrium of bodies acted on by three concurrent forces, to be solved graphically.
13. The calculation methods should then follow the graphic methods, and should be applied to numerical cases in which four-figure tables should be used, the angles 30°, 45°, 60° playing a very small part.
14. The Committee suggest that any of the following methods of dealing with the problem of parallel forces should be regarded as admissible:
    (α) By taking moments, assuming that a clear idea of the meaning of a moment and its mode of measurement have been obtained by experiment.
    (β) By the use of the funicular polygon, or by the equivalent geometrical device which deduces parallel from non-parallel forces.
    (γ) By defining a couple and proving from the parallelogram of forces that it is measured by its moment.
15. Both the graphical and analytical methods of determining the resultant of any number of coplanar forces should be taught, and numerical examples should be given.
16. It should be pointed out that all composition of forces assumes the existence of a rigid body to which the forces are applied, and that, failing the existence of such a body, composition of forces is unlawful and indeed unmeaning.
17. The impression, that the weight of a body is in reality a single force acting at its centre of gravity, should be guarded against. This and other cases where rigidity is assumed should be impressed on the beginner by contrasting rigid bodies with bodies which are not rigid.

18. *Machines.*—(α) The phrase "Mechanical Advantage" may preferably be replaced by the term "Force-ratio," viz., the ratio of the resistance to the effort when the effort is just overcoming the resistance.

(β) The consideration of work should be an essential part of the discussion of machines, and attention should at an early stage be given to "Velocity-ratio" and "Efficiency."

(γ) Systems of pulleys should not be referred to by numbers.

19. When the equilibrium of two or more connected bodies or parts of a single body is considered, the principle of "Separate Equilibrium" should be distinctly enunciated.

20. It should be clearly pointed out that all the results of statics apply to cases of uniform motion.

D. Kinetics (commonly called Dynamics).*

21. *Velocity.*—The meaning of the phrase "velocity at a point" should be carefully brought out by means of the idea of "average velocity." Average velocity should be defined as "total distance/total time," and should not be assumed to be identical with the arithmetic mean of the initial and final velocities, or with the velocity at half-time, or with the velocity at half-way.

22. Much stress should be laid on Newton's first law, and many examples given.

23. *Angular velocity* should receive attention, as in connection with it a great variety of interesting examples arise, *e.g.*, on the gearing of wheels.

24. *Acceleration.*—The velocity at any time should be represented graphically. This method should be used to illustrate the idea of acceleration, and the formulæ for uniformly accelerated motion should be obtained from the fact that the graph is in this case a straight line.

25. The formulæ for uniform acceleration having been proved as above, the fact that "the average velocity = the velocity at the middle instant" should be frequently employed in solving problems connected with such motion.

* The term "Dynamics" more properly connotes the whole science of force.

26. It should be expressly stated that all velocities with which we deal are necessarily relative to some base.
27. In explaining the parallelogram of velocities, it should be stated that we are proposing to determine the velocity of a point A relative to a base C from a knowledge of the velocity of A relative to a base B and of the velocity of B relative to the base C.
28. It should be permissible to treat elementary problems on the accelerations produced by forces by simple proportion
$$\left(\frac{\text{force acting}}{\text{acceleration produced}}=\frac{\text{weight}}{g}\right)$$
using the fact that a body's own weight produces acceleration g; and it should be allowable to postpone the consideration of mass until such problems have been discussed.
29. The poundal, when employed, should be used as an auxiliary unit, final results being reduced to gravitational units.
30. Atwood's machine should be regarded as a means of illustrating the laws of motion, and not as a practical method of finding g. The unsound method [mass moved $= m + m'$; moving force $= (m - m')g$] is to be condemned.
31. With the idea of preventing the notion that acceleration is always uniform, and having regard to the importance in physics of simple harmonic motion, it is advisable to consider such motion and the pendulum at an early stage, before the difficult parts of the work on projectiles and before oblique collisions.
32. Easy problems on the motion of a fly-wheel should form part of a course on elementary mechanics.
33. The phrase "centrifugal force" should be abandoned.
34. It should be clearly pointed out that the principle of the "Conservation of Energy" is not capable of universal application to mechanical problems, whereas the principle of the "Conservation of Momentum" can always be applied.
35. It should be pointed out that all the parallelogram laws are cases of a single law, that of the addition of vectors.
36. There should be no objection to illustrating the idea of a rate so as to lead up to the elementary ideas of the calculus.

E. Order of Teaching.

37. A short course of easy numerical trigonometry should precede the theoretical study of mechanics.
38. While several suggestions as to the order of (*a*) Statics, (*b*) Kinetics have been made above, the Committee does not wish to recommend that Statics should precede Kinetics, or *vice versa*.

Mr. W. H. Macaulay: Professor Perry begins his introduction of a boy to elementary mechanics at an early stage in his education, and the scheme which he has sketched appears to me to be a most attractive one. I agree with most of the principal points in connection with the gradual introduction of a boy to the subject: for instance, I agree with the taking of statics before dynamics, and taking it from a variety of points of view. There is one point in regard to which I can speak from personal experience, and that is the method of beginning statics with what is practically the principle of virtual velocities, having learnt the subject in that way myself. This may be made very interesting to an absolute beginner, and may be gradually linked on to other things. I also agree with Professor Perry on graphical statics. The subject is one of dodges, very good to learn if you want to use them every day, but it ought not to take up a great deal of the time of the student who does not want to use them for practical purposes. I also do not object at all to the use of the term centrifugal force, properly understood as not obeying Newton's law of motion. If a boy can get a gradual introduction to mechanical ideas, so much the better, but in any case he has some day to tackle a more systematic presentation of the subject of the principles of dynamics, and I wish to make a few remarks upon this, upon which I do not think Professor Perry has laid sufficient stress. What I wish to do is to plead for greater definiteness in the enunciation of the theory, and a closer connection than is usually found in the text-books between this theory and actual facts.

I do not wish to sacrifice any accuracy, but I believe that those who are attempting to master the subject often meet with

difficulties which are not inherent in it, and which hamper them unnecessarily. No one can master anything without thinking about it independently, and I should hesitate to smooth down any road which is laid out to students in such a way as to discourage independent thought. But there are certain things which we have all had experience of as to which, when we come upon them by a laborious and circuitous route, we feel that surely, if this be true, we might have been told it at once; and I think cases of this kind occur in the study of dynamics. There is too much tendency to mask what is really known of the subject under a mass of somewhat complicated relations between things which might to a greater extent be each hung on its own nail. First, as to the measurement of time, I would define this simply by reference to the clock—a compensated clock, of course—the principle of the clock, if you like; and surely this is what is practically taken as the measurement of time. Is it possible to imagine that, so long as no inconsistency is found in the measurement of time by repetitions of all kinds of operations, a thing that we are constantly relying upon in physics, we could be willing to give this up in favour of any other definition of time such as is sometimes given in Newton's first law of motion? I believe the pretended definition of uniform time by the first law of motion to be not only difficult, but altogether out of place and misleading. Everybody readily understands the principle of the clock, and this can be gone into by itself. It hangs on its own nail, apart from the laws of motion.

Next as to the base relative to which the motions referred to in the theory are to be reckoned. Why not deal with this point quite clearly and definitely? A suitable base is necessary. The laws of motion, in whatever form we put them, imply this. I have found it convenient as an introduction to that point to suggest the consideration of a bead on a wire stretched lightly between two legs of a table which can be pushed about the room. That sometimes suggests to a boy the fact that motion in a straight line involves some further considerations than merely the drawing of a straight line. Any reference to motion in a straight line implies the choice of a base, but surely a beginner ought not to be started on a quest of seeking a base for himself and identifying it with reference to actual bodies. This should be done for him

in the first place at any rate. We have a definite base for relative motions in the solar system which is practically perfect for that purpose, and the nature of which may be fairly easily comprehended. That for the dynamics of the whole universe we may require a base relative to which this has an acceleration is a perfectly easy thing to understand. It fulfils the conditions for clearness as being a thing to be thought of by itself, having to do with known objects. But I think it is best in teaching the subject to establish the position of the earth as a base at once. And again, let this be done in the most definite fashion, first in general terms that for ordinary terrestrial motions it serves so well that delicate apparatus specially arranged for the purpose is needed to show deviations from accuracy. This is satisfying enough to start with. At the same time some of the familiar cases of motions on a large scale which show the inaccuracy of this base may be pointed out, such as the tides and the trade winds; and then the way is prepared for dealing with some other cases which show the character of the approximate theory of motion relative to the earth, such as the centrifugal force component of gravity, Foucault's pendulum, and the deviation of a falling body from the plumb-line, and so on. There are, of course, cases in which the earth serves with perfect accuracy as a base; for example, collision, sudden changes of momentum, any base with finite acceleration will serve. But take one of two colliding bodies for base, and we get into difficulties.

Next as to forces. Of course, the theory reduced to its bare bones may be put in some such form as that with a suitable base of reference masses may be assigned so that their accelerations may be analysed into pairs. But it must be wrong to present the theory to a beginner so that he gets the idea that he is to start upon a quest for forces. The common introduction of force merely as defined by mass acceleration seems somewhat to mask the position in the subject of Galileo's great discovery, viz., that the feature of motion which the surrounding physical conditions determine is acceleration. This ought to be dwelt upon as giving us a reason for the definition. But as to the search for examples of force, the beginner should understand at once that he is not to undertake this. They have been sought out and classified for him. The classification may be subject to

change, but as it exists at any time it is the result of centuries of physical investigation; and the list is a very short one—contact, Newtonian gravitation, electrical and magnetic forces, and so on. This classification is one of the points which stands by itself, one of the keys to the situation, the fact to be learnt at once, the details to be gradually filled in. Then comes the result that mass is the same for all forces—in fact, a property of bodies. This, of course, is another definite point to be hung on its own nail. Familiarity with mass is, of course, to be found by experiment.

These, I think, are the points that I wanted to lay stress on in connection with the general theory. May I add a word on the subject of the engineer's unit of mass which Professor Perry advocates the use of—it has the great defect of having no name.

Professor PERRY: It is called "The Slug."

Mr. MACAULAY: If this name can be introduced, let us introduce it. I am not sure where to find a precise definition of it. But I must say that my experience is somewhat different from Professor Perry's. I think that it robs mass of some of its simplicity, and my experience, having had to do with a great number of engineering students, is that it leads to more mistakes in the way of omissions of g than the other system. I remember long ago being impressed by a question asked by a student, an engineering student of fair ability. He had been making some calculation about a rotating body, and he said: "How can it be right that the answer should involve g; surely the result would be just the same on the moon?" I think that this implies a very just criticism of Professor Perry's system. Another objection is that the use of g pounds as unit of mass is not analogous to the method used in the C.G.S. system. On the other hand, I admit that we have the convenient numerical relation between kilowatts and horse-power, which bridges over the gap which would otherwise make confusion.

Professor BOYS: I should like to offer a few remarks upon this interesting paper which Professor Perry has given us. It is of course a case in which those who wish to join in the discussion would have been better placed if they had had the opportunity of seeing what was printed before coming into the room. Having

only seen this at the moment, it is of course impossible to go through the paper as a whole and offer remarks upon it paragraph by paragraph. It will be much better for me, speaking as one thoroughly imbued, I believe, with the mechanical sense or sense of mechanical principles—though I will not undertake to say that I understand mechanics as Professor Perry understands it—only to refer to one or two points where I am either in agreement or disagreement with Professor Perry. I do most thoroughly and absolutely agree with the principle which he enunciated, viz., the essential desirability of dealing with fundamental principles and not worrying too much about details. I think that story about the competition that he had with the other teacher and that examination is a beautiful illustration of the advantage of dealing with essential principles and not worrying about innumerable details. I may perhaps relate a story illustrating the same point. A friend of mine in London, who was a pupil of Lord Kelvin's, when attending one of the latter's lectures heard him say: "And now we come to the principle of the lever. You will understand that levers are divided into three orders—levers of the first order, levers of the second order, and levers of the third order—but which of them is which I cannot for the life of me tell you." This story will perhaps illustrate what I mean when I say that text-books were at one time filled up with absolutely futile and unnecessary kinds of discrimination which had nothing whatever to do with the subject. Lord Kelvin, in spite of his statement that he did not know which was which in regard to the three kinds of lever, probably taught those pupils more about levers than another teacher who wasted time on details which are not essential to the grasping of the principle.

The word "poundal" has been introduced. My experience is that the poundal is the most terrible trap ever set for the unwary student. The rubbish I have had written by candidates at examinations in mechanics where they are dealing with the pressure exerted by a screw, the futility of it all, seems to indicate that the poundal does more harm than good. I object to it altogether. I quite agree that centrifugal force, in spite of all that has been said as to its iniquity, is a good expression and carries a good wholesome mechanical meaning to a student. I see no objection to it whatever.

Now we come to a part which I don't feel very able to speak about, viz., what can a boy do and understand at the age of thirteen and fourteen years, and so on. Of course the schoolmaster in general, seeing these statements about what a boy of twelve or thirteen or fourteen is going to understand, would be entirely aghast, but that is not the point. What I want to urge is this: that the mechanical model is one of the most valuable educative appliances that we possess. Devices of all sorts can be obtained for a few shillings. They may be used simply as a means of amusement, but a few well-directed words when these are being used, not upon detail, but upon fundamental principles, will I believe enable a boy to acquire the fundamental ideas of mechanics in a better way than he would do if he were merely crammed out of text-books. I affirm, for instance, that if you give pulleys to a boy and let him play about with them, and find out that with a suitable arrangement he can make little weights pull up big weights, you will interest him, and at the same time he will learn certain fundamental principles which he would not acquire so readily in any other way. There is a great deal more which one would like to say, but I have only been able to express myself very imperfectly on one or two points. Had I had the paper before me yesterday I should have put together something more consecutive and logical.

Professor BRYAN: I am rather reminded by this discussion of the story of the man who fee'd the guard of the train by which he hoped to make a connection at a distant junction. He missed the connection, and proceeded to blame the driver of the engine, who said that the traveller had "greased the wrong end of the train." At the present time we have men who are very competent to work out research in mechanics, who have high university degrees which have cost a good many thousands of pounds, but who are paid totally inadequate salaries for teaching the merest elements of mechanics to boys, and the teachers themselves have to pay taxes in order that these boys may be taught at their schools. Then, on the other hand, where the grease comes in is that there is too much encouragement given to boys who have no special aptitude for mechanics, but who learn the subject merely in order to gain scholarships by passing

examinations in it. I should like to see teaching more confined to subjects for which boys have a special aptitude. I would much rather that mechanics were taught to smaller classes, instead of our underpaid teachers having to drive these ideas on force and mass into the heads of a large class of boys. The necessity of boiling down methods of teaching to suit such classes deprives boys with real ability of the power of pushing ahead of the rest. This same remark applies to other branches of education. I wish there were no such encouragement for large classes of boys learning a large number of subjects for the sake of scholarships and temporary gains. We should get on very much better if the teachers were allowed more freedom of teaching, and allowed to adapt their methods to the requirements of boys who had a real aptitude and a liking for the subjects taught. It is no use expecting everybody to have a liking or aptitude for mechanics. There are some to whom it would be distasteful, and who succeed better in the study of languages; and no reforms of methods of teaching will alter this desirable diversity. What we need to do is to give much more attention to the learning and less attention to the teaching. We must not always blame the teacher if the boys do not do as well as they might. We should blame the system which overloads the competent teacher with a lot of dull boys, and which also leads to preference being frequently given to incompetent and badly qualified teachers. Long before I ever thought of learning mechanics my grandfather told me a few things about a lever, and he made it pretty clear to me that in a lever what was gained in power was lost in speed, and from this property I gained a good many sound ideas long before I could possibly have understood what was meant by defining force as the rate of change of momentum. The idea of mechanics which appeals most readily to a young boy is that it has something to do with machines, and that machines have something to do with turning out useful work; and there appears to me to be no better way of stimulating interest in the subject than showing the beginner that when you have got your machine for changing one kind of work into another you are no better off than you were when you started. With regard to the question of units, paragraph 28 gives us the best way of treating dynamical problems in the earlier stages, since it allows of this

question being shelved until the student has become familiar with the principles of the subject. I understand that Professor Perry advocates some kind of engineers' unit of mass which is in some mysterious way defined by W over g. I have never been able to understand the use of putting M equal to W/g, and a definition which makes the mass of a body depend on the units of length and time, and also makes it vary with the locality, can scarcely be intelligible to a schoolboy. It certainly is not intelligible to *me*.

The whole trouble about units arises from the attempt which modern writers have made to improve on Newton's good old second law by substituting "equal" for "proportional." Then again I doubt whether it is much use teaching *average* boys book-work about the link polygon and linear displacement. They will learn the bookwork off by heart, and they will reproduce it much after the style of a guide describing a historic ruin. If any slight modifications are asked for in the proof they will begin at the beginning and repeat the story exactly as they learnt it, utterly disregarding the absurdity of their answer.

I don't think Professor Perry is right in blaming the *examination* system. It is not the examination system which is to blame so much as that fetish of the British Board of Ignorance, commonly styled the Board of Education. I refer to the syllabus system. Abolish the syllabus, give the teacher perfect freedom of method, and let him send a statement of what he has taught to an examiner, and you can leave the examiner to frame suitable questions to test the pupils' knowledge. I have seen answers sent up by candidates examined in this way, and the results I have considered were perfectly satisfactory. The syllabus system is essentially a British institution. It arises from the fact that the Britisher does not know a good teacher from a bad one. He therefore distrusts both, and invents a syllabus in order to help the bad teacher to teach on insufficient knowledge of his subject. The Britisher prefers to appoint as teachers babes and sucklings who have spent only two or three years in trying to learn the whole of science at a technical or training college. The experienced teacher is excluded from many appointments by an age limit, and left to go to the workhouse. I think it is a pity that friction is left so much out of account in elementary mechanics. Mechanics as at present taught—especially kinetics—is unreal, and what we want is to make

the subject much less cut and dried. But here another difficulty arises in England. I was setting a mechanics paper not long ago in which I tried to get practical illustrations for every question, such as the horse-power that is carried past any point by a river flowing with given velocity, or the resistance of air to airships. The questions tested the candidates' power of obtaining results that could be understood and interpreted in the light of common sense, but they were much harder questions than they ought to be because they all involved an amount of arithmetical drudgery which was no test of the candidates' knowledge. This drudgery consisted in the mere multiplication or division by 2240 or 33,000 or 60/88, or 5280, or other numbers arising out of our artificial system of units. If we had the metric system the results could have been attained without this troublesome drudgery, and what is more, the work would have been a test of the candidates' power of manipulating ones and noughts, which are the two numbers responsible for the great majority of bad schoolboy "howlers."

Professor Hicks: I don't know that I have many words to say, but I should like to say that I have listened with the greatest interest to what Professor Perry has said in his address. There are one or two points, however, to which I should like to draw attention. The first is the remark about the insufficiency of the knowledge of boys when they come to colleges. I had a great deal of experience myself in dealing with pupils from all sorts of schools who came to the college of which I have been Principal, and I am sorry to say that the highest schools are the worst from the point of view of teaching mechanics. The boys from these schools come with no knowledge of mechanics. I find that they have no mechanical ideas at all. Boys who come from the elementary schools and have passed through their higher secondary schools or central schools generally are very good, but boys who come from public schools are very bad. And I think if we could do something by which we could make the teaching of a certain amount of mechanics—that is to say, of mechanical principles—obligatory in all schools we should do a very good work. In that respect I differ from Professor Bryan, who says you can only teach boys those things for which they have a special aptitude. To carry that argument to a logical conclusion,

we can only teach arithmetic to boys who have an aptitude for it, and the same with reading and writing. Next we come to the point of how we should teach elementary mechanics, or the method to be employed, and here I am entirely in agreement with Professor Perry as to the teaching of principles, and not getting up a lot of rules. The principles are very few, and if you get a thorough impregnation of the principles you can solve most problems. Those that you cannot solve are those which depend on mathematical calculation. As to the way in which those principles should be taught there may be a great variety of opinion. There are many ways of approaching a subject, all of which may be perfectly right. Very much depends on the teaching. My own experience is in favour of approaching this subject from the kinetic point of view. First let them find out by experiment that momentum remains constant. Of course, the first thing depends on what mass is; then we must proceed to show that when two bodies collide with equal velocities they come to rest. By making experiments of velocities of colliding bodies they get to realise that momentum remains unalterable. Given two colliding bodies in a straight line, the momentum lost by one is gained by the other. By getting a large number of experiments students come to a realised knowledge of that. It is then easy to pass on to the idea of force. The question of the poundal has been raised, and there I am afraid I am in opposition to Professor Perry. I think the poundal is a most useful institution. Of course, people can talk a lot of nonsense about it. If they did not have the poundal they would talk nonsense about something else, but I think that we want to get people to use absolute rules, and I have always found it extremely useful to have the two units to go to—the dyne and the poundal. One of the great difficulties of students in dealing with mechanical problems—I am speaking of the ordinary student—is that they will not realise how simple the mechanical question is. They think it is something up in the air and out of reach, and they will not apply common-sense reasoning to it.

[Added November 20, 1905. I notice that Professor Perry cannot admit that the "difficult subject of kinetics can ever be taught before or even along with statics." It may be answered that this has been done, is being done, and will continue to be

done by a large number of teachers. I hold that the development of any subject before a student should always proceed in logical sequence so far as it goes, and that the first ideas presented should be the simplest possible, and those requiring as little preparatory road-making as possible. Velocity in a straight line and its measurement is familiar to every beginner from his childhood. Consequently he can start at once experimenting, and learn—and realise—his first fundamental facts: (1) To measure the relative masses of bodies, (2) the conservation of momentum in one direction. Everything he measures he can see, and the ideas are simple. If he begins with statics, the first ideas are complex and sophisticated—a force is a thing which cannot be seen or realised when bodies are at rest. The student is led to think of force as something which resists his will to move a body—an idea which is the cause of endless confusion in his mind and the origin of the dynamical nonsense and obscurity of view so often displayed by correspondents in the engineering journals.]

Dr. Blieden: Speaking from the point of view of a student of mechanics, I should like to say that in trying to understand the principle of mechanics I met with two great difficulties, and I dare say other students have experienced the same difficulties. The text-books do not state definitely enough the difference between work and energy. Work is very often spoken of as a form of energy, but as a matter of fact that is fundamentally wrong and leads to a confusion of ideas. Work is a process, energy is a faculty. Energy is the result as well as the source of work, but work is not energy, nor is energy work. If you once differentiate between work and energy, you want a wider definition than is furnished by the text-books. Work can always be defined as the product of alteration of space or rate of motion by another factor which corresponds to force. This may be linear alteration, in which case we have linear force; it may be alteration in volume, implied by pressure; it may be change of surface, implied by surface tension; it may be electrical work, the displacement of a definite quantity of electricity. And the text-books sometimes speak of electrical energy when what they ought to speak of is electrical work. Before stating my second difficulty I should like

to read Clause 34 of the report of the M.A. Committee on the teaching of elementary mechanics. That report says you must not pay so much attention to conservation of energy, but to conservation of momentum. The principle of the conservation of energy only deals with one aspect of natural phenomena, and leaves entirely unnoticed the second aspect, which is equally as important as the one with which the conservation of energy deals. That second aspect was not the conservation of momentum, but the principle of the dissipation of energy. If mechanics simply, in teaching the theory of force, takes account only of the conservation of momentum, it fails to take account of the second aspect of energy, viz., what Lord Kelvin calls the dissipation of energy. In mechanics we can take account of both of these principles if we divide force into two kinds, reversible and irreversible. A reversible force is the weight of a body. You do work in lifting the body, and in returning it does work. The pressure of an elastic liquid is of the same kind. You do work in pressing the liquid, and as it returns to its original state the liquid does work. Friction is a result of irreversible work, because it produces heat, which cannot be reconverted into work.

Professor Bryan: That is exactly synonymous with what are rightly or wrongly called conservative and non-conservative forces.

Dr. C. H. Lees: I think, in fairness to those here, I ought to preface my remarks by saying that my experience is derived mainly from teaching students preparing for matriculation examinations and examining these students, and from inspecting secondary schools in which mechanics is taught. As the result of this experience I take a rather more optimistic view of the present state of teaching, certainly in the North of England, than Professor Perry takes. I have found in the secondary schools that a very fair proportion of the teachers of mechanics are actuated by the same desire as Professor Perry. They are anxious to teach the principles of the subject in such a way that their pupils will be capable of applying them to new problems, and that mechanics is not to them a mere collection of formulæ to be forgotten as soon as they leave school. Amongst the teachers I found, too, a deprecation of "dodges" which are applicable to one particular type of problem, but which absolutely fail when

applied to another type. They are anxious to teach principles and general methods. I may perhaps point out that this desire to teach general methods is to be found, too, in the higher branches of the subject. The most recent Cambridge text-book on mechanics adopts general methods, and gives none of the "dodges" so often found in the books which preceded it. Amongst teachers in secondary schools there is a genuine desire to find the best method of teaching the subject, and the question whether the teaching ought to be practical or merely blackboard work has been considered by them. I have not found a single teacher who did not believe that the proper method is the practical one. The reason why each cannot adopt it is that the funds of the school do not allow them to purchase the apparatus. If the apparatus is to be purchased it must be simple and inexpensive, and the objection has been raised that if the pupils work with simple apparatus the results they obtain are only rough, and they come to believe that the laws taught them are not accurate. To teachers who have brought that forward as an objection I have generally recommended that they should begin with simple apparatus, and should explain to their pupils that the apparatus has certain defects, and should show that as they remove the defects by improving the apparatus they get better results. Each defect removed brings the result more into agreement with the law. That in my opinion is the way to overcome the difficulty of using simple apparatus which only verifies the laws of mechanics approximately.

Sir David Gill: I confess to the fact that I never taught anybody mechanics, I never examined anybody in mechanics, and that of all men I am least qualified to speak upon the teaching of elementary mechanics. I can therefore only speak upon the subject as the result of my own experience in being taught. And I confess that I wish now I had been born fifty years later, and could have been taught mechanics by more rational methods and relieved from things which I did not sympathise with in an earlier stage of my career. I do believe that there is latent in many people the capacity for mechanics, because, at least in my day, everything was done to dissuade one from the study of mechanics, and one was *nolens volens* compelled to learn Latin rules and many things with which one had no sympathy. I

confess, on the other hand, I have known many fellow students who had no capacity whatever for mechanics, and on whom the teaching of the subject would be wasted. But I am quite sure of this: that with sympathetic teaching the larger cultivation of more men is possible, and I think to make men practical in this world, to make them useful members of society, the elementary knowledge of mechanics is at least desirable, if not absolutely necessary. As to how to go about it, I sympathise entirely with Professor Perry's view that men ought to be taught something about the practical laws of mechanics. One of the speakers has illustrated the importance of grasping the principle by the story regarding the lever. If the student knows he can use the lever in three different ways, he has the principle of the thing. With regard to the misuse of definitions, I remember Clerk Maxwell illustrating this by a funny story. He said he went into his room one day, and there was a white cat there which jumped out of the window. He and his friends ran to the window to see what had become of the cat, and the animal had disappeared, no one being able to solve the mystery. At last he solved the problem. He said it must be this. The white cat jumped out of the window, fell a certain distance with a certain velocity, and collided with an ascending black cat. There were therefore two equal and opposite cats meeting with equal and opposite velocities, the result being no cat. Without a proper understanding of definitions of these things one might arrive at such an absurdity as is illustrated by this story. I think that the great thing to be aimed at is a thorough knowledge of a few great principles. Given a thorough knowledge of those principles, mechanics is very largely the application of common sense.

The PRESIDENT (Professor Forsyth): Perhaps I may be allowed to say a few words; and what I shall say will be rather based upon my experience, extending back over a considerable number of years. I have now nothing to do with teaching elementary or advanced mechanics, but I know a little about the subject from my pupils, who belong to what I am happy to think is a new generation of teachers. What is wanted in the first instance is to accustom students to deal with things the properties of which they will have an opportunity of argument upon later.

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The first stage is not the stage in which students have to prove or attempt to prove anything, or be expected to prove anything. That belongs to a later stage. What you do want in the first instance is to accustom them to the ordinary relations of bodies and of their properties, and afterwards you can proceed to attempt to give some definitions that will be more or less accurate; but do not begin with definitions—begin with the things themselves, and let them know what they have to argue about afterwards. With regard to text-books, I will tell you the way in which I was taught—if it were teaching—the principles of elementary mechanics. I was the only boy in that particular class who was supposed to be prepared for mechanics. I was given a book and told to read the first chapter. Fairly obedient as I was then, I read that chapter. I was asked if I had read it, and on my saying "yes," I was told to read the second chapter. I read the first four or five chapters, and was asked my general opinions. I had none. The statements contained in the book conveyed no meaning to me. I did not want at that stage to prove anything. I wanted to learn something about which I was to talk afterwards. I was merely given a book, made to read it, and I learnt nothing. I am sure there have been hundreds of boys, at any rate in years that are gone, whose beginning in mechanics was precisely of that quality; and that explains why, when they came to teachers of mechanics who had something to teach them, the latter found that there was no basis on which to build. The manner of making new text-books has been to reproduce all that was in the old with just a little more, the result being that there was made a perfectly unwieldy thing containing surplus and inaccurate information. Now and then we do get a positively new text-book, and the new Cambridge text-book on mechanics deals with things in a way differing from the methods of older books; the man who wrote that book has had large experience in the teaching of mechanics in Canada. Now one word with regard to examinations. Of course, we are always told that they are necessary, but I live in a place where examinations seem to be the breath of life, and where they are utilised for the purpose of arranging students in an artificial order of merit, and in order to see who is the best gymnast in overcoming obstacles set by people who had nothing to do with the

teaching. That is examination run mad. What examination ought to do is to give a reasonable means of testing the students in their various classes, testing them to see that their work has been properly done. When examination is made to do more than that it is turned to an entirely different purpose, the main purpose being, as I have said, to put students in an artificial order of merit which serves no useful purpose.

In answer to request, the following remarks were communicated:

Sidney H. Wells: No one whose opinion is worth anything will, I think, attempt to deny that to teach mechanics in schools on the lines supported by Professor Perry would be of immense advantage to the boy in his further study of that and cognate subjects. But is it possible? Under present conditions I very much doubt it. In the first place, we have not the teachers capable of doing it. The past ten years have, it is true, seen a marked advance in the teaching of mechanics, from the old blackboard and diagram method to the individual experiment in a laboratory, discovering or verifying every law, with good workable apparatus and machines, and with the drawing-board or squared paper always at hand. But only those who know what is being done in this way know also how much there still remains to be done, and I repeat that there are not sufficient teachers possessing the training necessary to treat the subject even in this way, and certainly not in the more excellent way of Professor Perry's paper. Nor do I see evidence of any attempt to supply the deficiency. It may be urged that at present the supply is quite equal to the demand, although such has not been my experience; but I am quite sure that as soon as headmasters see that mechanics (using the term widely) can be made a much more living and really educational school subject than at present, they will be ready to give it a larger place in their time-tables.

Another difficulty is the expense of the necessary equipment, not to teach mechanics experimentally and practically, for that can be done to a large extent quite cheaply, but to do so in the manner indicated by Professor Perry. Secondary schools, still less elementary schools, cannot be expected to provide an equipment to include "all kinds of motors," "electric motors," "wheels

of turbines," or "centrifugal pumps," such as the paper refers to. The fact is that the mechanics of rotating bodies does not lend itself to simple experimental apparatus so that a class of twenty-six to thirty can be worked at the same kind of experiment at the same time, and this is why it is neglected. Apparatus of a fairly cheap and accurate kind can be obtained for experiments on the resolution of forces, friction, machines, elasticity of materials, &c., and what is now wanted is a list giving methods of construction and use of simple apparatus for the other branches of mechanics. If Professor Perry could do this a very real service would be effected.

And yet another difficulty is to see how time is to be spared in the ordinary secondary school time-table for the study of mechanics in the way advocated in the paper. I do not think that Professor Perry would desire to see the boy give less time to other subjects of a good education, and I am sure he must know how woefully boys leaving school at fifteen to sixteen are lacking in knowledge of subjects which come before mechanics. I would far rather that a student entering for a day technical college should be able to express himself in decent English, and to be able, say, to write an intelligent and observant account of a works he might visit, than that he should know how to differentiate and integrate X^n. At present we have to be satisfied if the lads of fifteen to seventeen straight from good secondary schools entering our technical day college can write a readable essay and work elementary problems in algebra, and it would have been impossible for them to have done the work suggested in the paper. Nor can I go so far as to approve of Professor Perry's statement that his remarks are applicable to the training of all boys. The object of school education, as I am now fond of conceiving it, is to enable the boy to find out how he can best express himself, and to train him to do so in the best way. All boys do not express themselves in the same way, and it looks as though Professor Perry thought so much of the supreme importance of the calculus to engineers and physicists that he would have everybody learn it, no matter what their future occupation might be.

My teaching experience of over fifteen years in a great public school, a university college, and now in a polytechnic leads me to the conclusion that the weakest part of our school training is

still in the mathematical and science subjects. The result is so eminently unpractical. And how can one expect otherwise when in 30 per cent. of the schools experimental and mathematical mechanics are taught by different masters? But I am optimist enough to believe we are moving forward and along right lines —if not so quickly as Professor Perry and others would wish, at least we are making real progress; and if some of the difficulties I have referred to could be removed, the result of the next inquiry of the 612 secondary schools would be very different from the last.

A. W. Siddons: Whenever I read an address by Professor Perry on mathematical teaching in schools, I always feel a desire to attack him; this feeling is perhaps aroused because the boys he speaks of are so much abler than 99 per cent. of the boys I have to teach. But on second thoughts I often feel that he is clamouring for a loaf where I should only have the pluck to ask for a crumb, but a crumb of the same loaf.

Here is an instance—let us first consider the crumb: The calculus is generally postponed to a needlessly late stage of a boy's mathematical education; this makes much of the elementary part of mechanics dull and heavy (perhaps it is tradition and examinations which forbid us to use the most suitable tool). Now, what is Professor Perry's loaf? At the age of fourteen he would like a boy to have a really good working acquaintance with the calculus and a fairly general notion of the various forms of energy, &c., and then he can get on to the theoretical study of kinetics. When is he to begin the calculus, I should like to know? At ten? I very much doubt whether one boy in a hundred is ripe enough even at twelve to begin it. The calculus is a powerful tool, but a dangerous one in untrained hands.

If I may venture to say so, the Mathematical Association Committee's report often asks for the crumb which can be swallowed and digested, where Professor Perry asks for the whole loaf, which would choke many teachers and leave them worse off than before. That report does not profess to recommend more than ought to be practicable under existing circumstances; Professor Perry probably thinks it does not go far enough, but it is pleasing to find that he is in general agreement with it.

There is one point I should like to lay stress on: mechanics examples set for beginners ought to be carefully chosen to illustrate the main principles of the subject. It frequently happens in text-books and examinations that the questions set are algebraical or geometrical puzzles, with about one line of mechanics in them. These are presumably the invention of the very mathematical man on whom Professor Perry pours not unjustified contempt.

In conclusion may I recommend to the notice of teachers of elementary mechanics the report * of the Mathematical Association Committee—a committee of teachers.

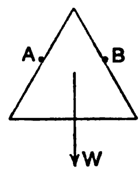
W. D. EGGAR: It is hard to be asked to contribute to a discussion when one has missed the trip to Johannesburg, but if any words of mine can help you and the cause of practical mathematics, I must do my best. I cannot quite agree with your views about the possible attainments of boys of fourteen. Even the clever boys who are capable of becoming good engineers are often late in developing. But I am with you entirely in desiring an earlier start in mechanics, so that boys may be ready for a college or university course by seventeen. Any elementary course for boys younger than seventeen must be experimental throughout. Indeed, to judge by results, it would appear that even with undergraduates the purely mathematical treatment is a failure.

It is perhaps natural that a mathematician should regard experiments with inclined planes and pulleys as tedious and obvious. For instance, he sets up with great care an inclined plane at an angle of 30°, and finds with exaggerated satisfaction that a roller weighing 200 grammes is supported by 100 grammes hanging over a pulley.

Somebody suggests trying 105 grammes or 95 grammes, and still there is equilibrium. This is annoying, because friction does not come till much later. He prefers to be content with a theoretical wheel and axle, in which a theoretical force supports a theoretical bucket without raising any water, and the subject remains dry. Of course, friction must come in early. Work and efficiency

* The report is published with the other reports of the Committee by Messrs. Geo. Bell & Sons. Price sixpence.

must be measured at an early stage, and here we have to do with forces not in equilibrium, so that I do not see how you can keep away from elementary kinetics at the start. Boys do not realise the direction of a force until they see the movement which it produces. A mathematical friend once showed me this diagram



made by a pupil. It represents a triangular card, of weight W, supported against a wall by two nails, A and B. An experimental treatment might have been useful here. But even where experiments are made, I find that the direction of a pressure or a tension is not realised until it is increased so as to produce a movement.

I should like to see a penny-in-the-slot automatic weighing machine in every passenger lift, so that this fundamental experiment, showing a connection between force and acceleration, could be within the reach of everybody.

As for the exact relation between force and acceleration, involving the difficult notion of mass, I think that might well come later. But I am sure that acceleration need present no great difficulties if the wave-curves obtained by apparatus such as Mr. Fletcher's trolley and paint brush are made use of.

As for the poundal, I am sure that, if you engineers do not like it, it ought to be abolished. We ought all to be pleased to make the acquaintance of the slug, even if we feel pained at parting with the poundal.

C. S. Jackson: I find myself in agreement with the views of Professor Perry, and confine myself, in drawing up this memorandum at his request, to some remarks on points of detail.

I. The present position as to the teaching of mechanics much resembles that which until recently existed as to the teaching of geometry. In the case of each science experimental facts furnish material from which certain inferences are tentatively drawn. In the case of each science by a process of abstraction certain

ideal entities are set up, and the inferences previously drawn may be shown by exact reasoning to follow unconditionally from the definitions of the entities involved.

The process of abstraction is most important, it is antecedent to the possibility of any truly scientific theory, but it is idle to ask a student to perform it without a knowledge of the concrete facts. Among the entities dealt with in geometry are the straight line, the plane, the circle. In mechanics—so-called rational mechanics—the material particle, the rigid body, the smooth surface present themselves.

The habitual omission in geometry of the experimental stage renders in many cases a student's conception of technical terms vague in the extreme. This is now universally conceded and has been remedied.

The omission of the preliminary experimental stage in mechanics has been far more complete than was ever the case in geometry. Had geometry ever been taught as mechanics has been often taught, we should have had editions of Euclid in which the cost of diagrams and the space occupied by them would have been saved.

It is needless to labour the point that to omit the entrancing historical incidents and the vivid practical details which the study of mechanics should incorporate is to throw away the means of rendering the study delightful. No one ever hears without pleasure of the riddle in which Hooke propounded his law, of the quaint English in which he developed it, of the simple practical problems relating to resistance which it solves.

II. That teaching aiming exclusively at success in an outside examination must be bad is now generally acknowledged.

The condemnation which appears imminent for examinations of this type should not be extended, however, to an examination by outside examiners in free, continual, and sympathetic communication with a teaching staff of students who have gone through a particular course of study. Such an examination possesses the real advantage of being very helpful to the teacher and the factitious advantage of offering a tangible guarantee to the general public, and very often to a governing body incompetent to form an intelligent opinion in any other way, that the work of the institution is being properly performed.

III. Profound assent must be given to Professor Perry's view that too often vital principles are obscured by surrounding the student with dense thickets of detail. No one ought to spend six months on any subject which is to leave him as it found him.

Without attempting completeness, some of the cardinal principles of mechanics, each one of which must leave a mark for life on the mind which has received it, may be referred to:

(*a*) The notion of variable velocity. A boy of twelve can grasp it, and as soon as he has done so fairly the differential calculus is easy.

(*b*) The notion of interpreting the area of a velocity-time graph as distance or the area of a force-displacement graph as work. Again not an obscure or complex idea, and yet one which illuminates the fundamental process of the integral calculus.

(*c*) The notion of measuring force by time rate of change of momentum.

Starting from $\frac{P}{w} = \frac{f}{g}$ we have $Pt = \frac{W}{g}.ft = \frac{W}{g} \times$ (change of velocity) $\left[\text{or, of course } \int Pdt = \frac{W}{g} \times \text{ change of velocity}\right]$.

The conservation of momentum follows at once from Newton's third law. The impulse of water jets on surfaces furnishes interesting and illuminating examples.

(*d*) The conservation of energy.

(*e*) The relativity of motion and the problem of how to measure the acceleration which is to be a measure of force.

A problem which does not intensify a student's grasp of some one of some such vital principles is probably a mere hindrance to his progress.

IV. As a member of the Mathematical Association Committee, I may be permitted to express satisfaction that Professor Perry gives a general approval to their report.

It must be remembered that such a report inevitably represents a compromise.

Many of us hoped to help in getting rid of that *βδέλυγμα ἐρημώσεως* the poundal, but liberty to use it was the price we had to pay for liberty not to use it.

No greater service could possibly be done to the cause of sound teaching of mechanics than definitely and finally to get rid of the

"poundal," accompanied by the ounceal, stoneal, tonal, and hundredweightal which logically accompany it.

V. The objection felt by many teachers to the term "centrifugal force" is that its use sins against the canon that a new technical term should never be introduced unless a new idea which cannot be expressed without the term is to be presented.

It is with deference submitted that the following statements contain all that is needful: *

(i) If the body moves in the prescribed curve it must have an acceleration inwards along the normal of magnitude v^2/ρ.

(ii) The actual external forces acting on the body must produce this acceleration—that is, supposing the body is moving at a constant speed. [Otherwise the tangential component of the resultant of the external forces must set up the tangential acceleration, and the normal component must set up the normal acceleration.] That is, the statical resultant R must be a force directed inwards along the normal of magnitude $\frac{w}{g}\frac{v^2}{\rho}$.

Further, the expression "centrifugal force" is apt to put a student on the wrong tack. It sets him looking for a force which will make the body fly away from the curve, whereas he ought to look for a force which is competent to make the body "take" the curve and prevent it from going on in a straight line.

JOSEPH HARRISON: Experimental work has now a recognised place in the school curriculum, so that it is scarcely necessary to insist on the great value of quantitative measurement as a means not only of inculcating a proper mental attitude towards physical phenomena, but also of acquiring a real insight into physical facts and laws.

The choice of proper units of measurement is an important one, and C.G.S. as well as engineers units will be used. As regards the unit of force, the pound is the most suitable one, on account of its convenient size, and because, in association with weight, the pound will have become ingrained in the youthful mind, and it is a unit in which he will speak and think. In laboratory work the pound force will frequently occur dissociated from gravity or

* No question of angular acceleration arises at this stage.

weight and in conjunction with other units, as for example, in pressures, tensions, stresses, work, potential energy, kinetic energy, strain energy, the mechanical equivalent of heat, &c. It is thus evident that if the boys are to think clearly the pound force should be an absolute measure, independent of geometrical position in space. This is easily effected, and it only requires it to be specified that the standard pound force shall be the gravitational attraction on the standard pound mass at a stated place, that is, *London*. The boys will soon realise that the errors incidental to physical measurements will seldom justify any account being taken of the variation of weight with latitude; but the important thing is for them to understand that the pound force, which enters into so many things with which they deal, is absolute and not variable.

The conception of force as the rate of change of momentum and the comprehension of Newton's laws in their vector significance will only be acquired slowly. A knowledge of vectors must first be obtained under less abstract conditions by experiments on the statical equilibrium of forces, and by problems on position, displacement, velocity, and acceleration vectors; also straight line or scalar dynamics will require to be experimentally studied. With regard to vectors, it has been found of great advantage to adopt some simple yet definite method of numerical specification to which boys shall get accustomed. Thus, a vector in a plane being defined by a magnitude a and a direction α, the latter being an angle measured (say counter clockwise) from some standard or zero direction, might be written (a, α), or a_α; the components in the directions 0° and 90° would be $a \cos \alpha$ and $a \sin \alpha$, and in the direction θ the component would be $a \cos (\theta - \alpha)$ or $a \cos (\alpha - \theta)$. A vector located in a line, called by some a rotor, such as a force, might have a specification like xa_α, where x is the intercept of the line of the vector on a line of reference in zero direction, measure from a point o in the line. The component in the direction θ as on $\cos (\theta - \alpha)$ as before, and the moment about o is $x a \sin \alpha$. Thus both graphical and numerical treatment of vectors can go hand in hand, and boys are provided with a simple means of giving complete instead of partial answers. At present a boy pays little attention to anything except the magnitude of a vector.

When the boy begins experiments in linear dynamics, the unit of

force to which he has become accustomed should still be the absolute pound. The derived unit of inertia or mass, acting on which unit force will produce unit acceleration, will naturally be 32·182 lb., and masses being multiples or submultiples of this unit might well be provided. If this unit of inertia receive any special name, such as "slug" or "nert," the name and the idea associated with it will be quite distinct from the name and the notion of a force. The inevitable confusion attendant on the introduction of the poundal force and pound inertia will thus be avoided.

An adequate conception of inertia is not easy of attainment, and it would be a distinct gain to call in the aid of the muscular faculties. Thus, instead of applying a force through the agency of a raised weight, let it come as a direct pull exerted by the hand, and measured by a light spring balance. A strip of steel 10″ long, $\frac{1}{4}$″ broad, $\frac{1}{32}$″ thick, bent into a circle, something like a cyclist's trouser clip, would serve admirably for measuring the force, for it would open out about 5″ under a pull of 2 lb., and its own weight would scarcely be perceptible, being less than $\frac{1}{4}$ oz. A good experiment for imparting a sense of inertia, and at the same time of making very evident the kind of force action which goes on in a simple vibration, is for a boy to move a fairly heavy mass backward and forward, keeping always in line with a guiding point which has a S.H.M. imparted to it.

Experiments on the statical equilibrium of forces, and on the principle of moments, might some of them be made on a horizontal board, using the trouser-clip spring balance, instead of on a vertical board using weights. The ideas would again be received partly through the agency of the muscles. Half a dozen such clips could be bought at a cost of sixpence, and so uniform in strength that a simple scale, graduated by the boy himself, would answer for measuring the pull recorded by any of the springs. Thus a whole class could be making experiments simultaneously, and any boy specially interested could carry his apparatus home.

Professor G. M. Minchin: The difficulty experienced in teaching such branches of science as dynamics and hydrostatics in the case of boys who have come from the public schools, and from others frequented by the same social class, arises from want of

early familiarity with scientific facts and thought. Our education is still of the mediæval kind, being dominated by headmasters who are almost exclusively clergymen of classical training. At present it is almost impossible for a scientific layman to obtain the headmastership of a public school; and so long as this remains true, the education of boys of the higher social classes will continue to be unscientific.

The mischief begins in the preparatory schools. These institutions have developed enormously within the last generation. They are miniature classical public schools, whose chief business consists in preparing their more promising pupils for the scholarships of the larger institutions. Scientific teaching forms no part of their *régime*, and for this the British parent is largely responsible. He is a hardened conservative, and because someone has told him that little boys cannot learn anything but Latin grammar and subjects of a literary kind, he has become convinced that the elementary facts and experiments of science are quite beyond the capacities of boys of nine or ten. Hence the usual five or six years spent in an endeavour to learn some Latin and Greek in the preparatory school, where reading, spelling, and English grammar should be the main work. For these homely subjects we now rely on the nursery alone, and the result is that they have practically disappeared. The average boy of the better classes comes now at the age of fifteen or sixteen to the study of mechanics and other branches of science with a mind wholly unaccustomed to scientific thought of any kind, and then he is called upon to acquire, within a very short space of time, a knowledge of hydrostatics, dynamics, and other branches of science, for the purpose of some examination. Hence hurry, absence of understanding of fundamental principles, and cram. It is almost useless to discuss the improvement of our text-book methods until the unscientific atmosphere of our schools is radically altered. The directing influence should be conferred upon scientific laymen; but of this change there is almost no sign whatever. The evil with which we have to contend lies deeper than the text-books. That is my opinion as regards the position generally.

I would add a few brief observations on some details in Professor Perry's paper. Where has Professor Perry met the boy

of fourteen who has "learnt elementary trigonometry, can use logarithms, knows what is meant by speed, can differentiate and integrate x^n, has many applications of the use of the calculus, has a fairly clear notion of the various forms of energy and their equivalent values, knows about the law of work, about friction, has experimented on the efficiency of machines and the loss of energy by friction, the efficiency of all kinds of motors," and so on? All the knowledge in all the public schools of England does not amount to one half of this total. That it might do so under better conditions is another matter.

With Professor Perry's words, "When understanding [of principles] is effected, there is no difficulty about the proofs. It is quite usual to find men who can prove everything without having any comprehension of what they have proved," I agree most thoroughly, because a vast amount of our testing of knowledge by examination consists of the unintellectual process of writing out whole pages of "bookwork," the value of which the examinee proceeds to demonstrate by making the most ludicrous attempt to *apply* it to some simple problem.

Finally, in opposition to Professor Perry, I hope to see the term "centrifugal force" utterly banished, for reasons which I have often given. It is responsible for more mischief in the minds of students and in text-books than any other term in science.

C. Godfrey: No doubt the engineering boy has the best chance of getting at the inwardness of mechanics, but the problem for most of us is how to teach the subject without engines. My advice to anyone who suspects his knowledge of mechanics to be academic would be—read Professor Perry's *Applied Mechanics*.

Statics is a fairly easy matter if one begins with experiment and goes on to graphical work, gradually adding algebra and trigonometry to the dish. Nor need experiment cease after the first stage; any school should be able to get hold of some bit of machinery with plenty of friction in it, say a screw jack, and investigate efficiency. Plotting "load" against "effort" leads to very striking results.

Kinetics is dreadfully difficult for the average boy. The

trouble seems to lie in the fact that the simplest laws of kinetics are contradicted by the boy's everyday experience. All the reforms that Professor Perry enumerates are now in working order (at least at some schools), but the difficulty of kinetics remains.

There is the question of mass and weight. In vain one resorts to the centre of the earth, or to Mr. Wells's cavorite projectile; it is all too hypothetical. Perhaps Mr. Fletcher's trolly experiment will clear up the matter. I remember, as a boy, being puzzled to understand how the weight of a train (acting vertically) could have anything to do with its acceleration under a pull (horizontal) from the engine.

We might give a touch of reality to the kinetics course by brake horse-power determinations. It should be possible to rig up for a few shillings a brake-drum on a motor (electric or water); even a motor-cycle on a stand or a foot-lathe might serve the purpose.

The plan of putting the class through the easier parts of Worthington's *Dynamics of Rotation* has had the happiest results. A little calculus helps, and is easily learnt. Dynamics of rotation does not pay for scholarship purposes, but is a pleasant change from the particles that slide down the focal chords of parabolas.

Engineers talk in a very confusing way about centrifugal force. When a particle moves in a circle uniformly the force on the particle is centripetal and the force on the constraints is centrifugal. But the popular use of language and the popular belief is that there is an outward force on the particle. The following quotation is from an engineering text-book: "This force, tending to cause the stone to fly outwards, is known as *centrifugal* force, and is the force utilised in the centrifugal pump, the water taking the place of the stone trying to fly away from the centre." It is the point of view illustrated above that mathematicians are fighting when they deprecate the use of the term centrifugal force.*

I am one of those who doubt whether the average boy ever

* Take the case of an element of fluid moving in a curved stream line. We convert the kinetic problem into a static problem by stating all the forces *acting on the element* normal to the stream line, and one of these forces is an imaginary centrifugal force. Distinctly I want it to be understood that it is *not* a centripetal force.—J. P.

understands that force is equal to rate of change of momentum, and I should be astonished if I found "vector change of momentum" brought home successfully to anyone but a mathematician.

W. W. F. PULLEN: Before the teaching of mechanics can become as uniformly good as its importance warrants, its position as a subject must become much more stable than it is at present. In some schools it appears under the guise of introductory physics, in others as mixed mathematics or elementary science, and again in others it is carefully dissected into theoretical and applied sciences. I have known it meet an early death under the title of workshop arithmetic. To judge from appearances, it must be regarded as a sort of intellectual pill which requires hiding in mental jam before it can be assimilated into the system.

Mechanical laws are the foremen who regulate and control the operations in the majority of natural phenomena; and on this account alone it should be given a position comparable with mathematics or chemistry or biology. But when viewed from the vantage point of the technical world, its importance is so pre-eminent as not to permit of discussion.

Professor Perry most wisely draws attention to the vital importance of emphasising principles, though his story may almost be read as to imply that exercises are unimportant, if he did not so consistently impress upon the readers of his text-books that they should be continually working examples.

Although operating over so vast a field, few realise how small is the number of laws or principles in elementary mechanics. The two laws of equilibrium, together with the proportionality of stress to strain, cover the whole range of statical problems, including hydrostatics, while Newton's laws of motion cover the remainder of the ground.

It is in his attempt to form a real mental image of these laws and in the application of elementary mathematics to them that the student often finds the bunkers are numerous and apparently difficult to get out of. Emphasis should be laid by the teacher upon the difference between a law and a definition. The former is discovered, while the latter is made.

I very much deprecate the pseudo-specialised study of *parts only* of mechanics for special examinations or technical work, and often without due relation to principles. In certain branches of technical work, graphic statics is supposed to be a sort of talisman which will secure the answer without the "unnecessary" and troublesome method of calculation. In such cases it is really a cloak for ignorance, and in one of these I had pointed out to me I found a teacher labouring through the vector and funicular polygons to solve a problem on the wheel and axle.

But to return to the teaching of boys. Professor Perry's suggestions are admirable, but how are they to be carried out? To make them possible, the head of the school must be in sympathy with them. Further, the Mr. Barlow who essays to teach the subject will have, not the ten or eleven pupils suggested by Professor Perry, but anything from twenty to five-and-forty. Most probably the said Mr. Barlow is engaged in actual teaching from twenty-five to thirty hours per week, after which he may have to correct as many as three hundred pieces of homework, besides devising and scheming apparatus at the minimum of expense. Over all this he is supposed to wax enthusiastic on a salary less than that paid to many an artisan.

I am afraid that the chief cause of the indifferent teaching of mechanics lies in the conditions under which it is taught.

Passing on to details, I object to commence the subject at the dynamical end, because a beginner soon gets out of his depth in dealing with more or less abstract quantities. To make progress, he must feel the ground he treads upon, however imperfectly; and experiment is the only means that will enable him to do this satisfactorily. As it is much more difficult for a beginner to experiment successfully in the presence of acceleration, I have invariably treated this late in the course, instead of beginning with it, as is the usual custom.

Professor LARMOR: I have read with interest your remarks on the teaching of mechanics. That at present much is chaos seems difficult to deny. It is also clear that if, as you suggest, each boy has an ideal "Mr. Barlow" all to himself, the enlightenment which you eloquently describe will come to him without great effort, and as an interest might even compete with athletics: his

outlook, if he is an average boy, would be expanded and stimulated, while no detriment worth considering would be done to his reasoning powers. Even vectors and motors could be made rational interests.

But the present condition of the actual world does not seem fully to admit of this. Classes are large and experimental illustration troublesome. The successful text-books are naturally those that make it easiest to handle large classes. They thus become abstract and dogmatic and unmechanical, and need periodical stimulation such as you are now supplying. Even "Mr. Barlow," left to himself, might, like the rest of us, take the path of least resistance, and instruct about words rather than things.

I do not join you in decrying examinations. As things are, I think rational examinations give most important guidance and stimulus in places where these are most needed, and often exert pressure in favour of improvement in the only way that is possible. I am even free to confess that I imagine that the most useful work I have ever done has consisted in the dull occupation of examining. If I had not thought so I would have done less of it; but I admit, on the other side, that I usually sailed as near the wind as I could in the matter of being tied down to a schedule. Even in the best regulated institutions the breath of outer air brought in by an extern examiner need do no harm, assuming that he is a man of sense and not an examination hack.

I cordially agree that formulas and algebra are at present in the main a snare, and should be kept in bounds by the method of examination and such other ways as are open. There is often a similar snare lurking in the current term "elementary." Not seldom an elementary text-book is one confined to an artificially limited range of ideas, but made as difficult as possible within that hampered domain.

I also agree that what you call the method of water-tight compartments is much overdone and is inefficient: this is sometimes shown, where free trade in these matters prevails, by the whole of the effective direction and instruction getting concentrated into the hands of the private tutor.

After all, there is something to be urged for the British plan of getting boys and youths into a correct groove and attitude of mind,

and letting them educate one another automatically. But, as I have admitted, there are difficulties. For example, I believe I know from instances that a text-book composed in a suggestive, stimulating way such as you describe would have a poor chance of coming into use, as involving too much drilling and supervision for actual arrangements.

Professor ARMSTRONG: I admire the manner in which my friend Professor Perry, while discussing the teaching of elementary mechanics, deals in his own inimitable way with the problem of education generally; and he is right in so doing, as the part cannot be separated from the whole: mechanics will be badly taught, and every subject will be badly taught, lay down what rules we may, until method in education comes to be regarded as a burning political question and either some aristocrat or an interfering person with ideas, like Mr. Chamberlain, appears on the scene as the advocate, in season and out of season, of a rational policy, and eventually forces its acceptance on the schools as the one and only possible means of salvation. We might with advantage borrow the German Emperor for the purpose for a year; but failing him we cannot do better than make Lord Curzon on his return from India Minister of Education with plenary power to enforce a policy. If people could only be led to see that proper education is the precursor of any form of successful trade—whether free or preferential—they might say to politicians, "A plague on both your houses! Give us bread, not stones; force the universities to do their duty to the country."

That the whole of our teaching is depressed by the failure of each grade in turn to do its work efficiently I agree. But I do not agree with Professor Perry that "the universities and technical schools use bad methods because the public schools fail to send up intelligent boys." It is not the intelligence of the boys that is in fault so much as their training.

Universities use bad methods simply because they will not give themselves the trouble to think out and use good ones—they do not realise that methods which were good once are now no longer suitable. Continued practice of academic, literary methods, the worship of precedent during several generations, has reduced the

body educational to a mass of carcinomatous tissue, leaving it all but destitute of observing and reasoning power.

Professor Perry has himself done a great work as a reformer, but his preaching would have been in vain, for the most part, had he not, in virtue of his office as an examiner for the State, exercised a kind of German Emperor's authority and built up an efficient practical mathematical fleet in spite of public opposition. His success shows how the thing is to be done, just as Admiral Fisher's success in introducing a new type of education into the Navy shows how our schools are to be reformed. Nothing short of compulsion will bring about the reforms we need in this country—we simply cannot wait for a natural development of intelligence among the teaching fraternity; time is too precious in the fierce race of competition for which the nations are now entered.

Professor Perry, I fear, has little knowledge of young boys, or he would not say, "I take it that it is not before the age of ten the average boy begins to reason for himself," &c. Why this grossly libellous statement should be so frequently uttered is difficult to understand: the idea must have arisen in the mind of some schoolmaster who, finding boys unable to appreciate what he regarded as reasoning, set them down as incapable of any form of reasoning. I believe the boy is not ten minutes old before he begins to reason, and that his reasoning powers continue to grow at compound interest rate so long as he is kept from school: then the curve begins to drop, and soon runs asymptotically close to the base line. When we realise that we must believe in intelligence as an innate possession of the majority, and seek to develop rather than to repress it, progress will have been made towards the introduction of a satisfactory system.

Everything in education is arranged at present from the point of view of the grown-up who has lost touch with childhood: we do not administer what is proved to be assimilable, but what we in our grown up wisdom think should be. Physiologists are fast coming to the opinion that we eat far too much nitrogenous food—it is time that educationists realised that we give far too much academic, classical, clerical food, which, like proteid, for the most part, has to be worked up merely in order to get rid of it again.

What I complain of in the boys who come to me is that they lack alertness and enthusiasm: they have neither common knowledge nor common sense; not one of them has learnt to read or to write about common things; worst of all, they have no initiating power. Swearing that Britons never, never shall be slaves, we yet allow our children to be brought up in habits of slavery and to do little more than do as they are told. Professor Perry says, "Initiative and originality are gradually disappearing from courses of instruction." I should go further and say—they are non-existent. B.Sc.'s are not even to be compared with buttons. Buttons will fit into holes and hold things together in a useful manner—but most B.Sc.'s won't fit into anything practical. And it is all because the Chinese system prevails among us in an ever-increasing degree—the literary men rule the roost. We may talk ourselves black in the face, but they will not understand us, let alone heed us: to speak of things practical is to address them in Chinese. Unless we force them aside, we shall do nothing.

As to Professor Perry's scheme, in my opinion it lacks the detail which is necessary to make a scheme go. It is all so simple to Professor Perry—he thinks that a mere indication of his meaning will be understood. But the average teacher needs spoon-feeding at present: it is no good merely to advise him what to buy; the Professor must himself buy and prepare the food and present it to the patient's very mouth. His success in reforming mathematical teaching is due to the fulness of the instructions he issued, just as the success achieved by the British Association in improving to some extent the teaching of elementary science is due almost entirely to the issue of a scheme which has served the purpose of a text-book.

Whatsoever scheme be devised, I trust it will deal primarily with common things—the see-saw, the scales, the crane, the pulley, the windlass—and that the object kept in view will be, through the study of mechanical appliances, to develop observing power and the critical faculty as well as the sense of exactness—all of which are lacking in boys taught under existing circumstances.

Professor Love: I am in hearty sympathy with what I take to

be Professor Perry's main thesis: that the study of mechanics ought to be pursued experimentally as well as mathematically. At the same time, it seems to me that too much importance may be attached to experiments with special apparatus designed to illustrate particular points in the theory. Much should be made of the dynamical interpretation of facts that are familiar in every-day experience. Otherwise students may get the notion that mechanics is a kind of boredom prevalent in the physical laboratory, just as, when experiments are dispensed with, they are apt to think of it as a kind of boredom associated with the mathematical class-room. The principles of mechanics ought to be presented, as Professor Perry says, from so many points of view as to become part of a man's mental machinery.

I should like also to express the opinion that some historical study of mechanics is extremely desirable. The principles are appreciated better when something is known about the efforts by which they were discovered and the state of ignorance with which the pioneers had to cope. There exist one or two good modern books in which the science is treated historically; but perhaps at present the best means of promoting this kind of study would be occasional lectures. Astronomy has long been a favourite subject with "popular" lecturers. Could not mechanics be made equally attractive?

Professor WORTHINGTON: Through the kindness of Professor Perry my attention has been drawn to the report of the M.A. Committee on the Teaching of Elementary Mechanics, which was the subject of some remarks by Professor Perry himself at the last meeting of the British Association, and I have been invited to contribute to the discussion which he initiated.

The report appears to have been drawn up for the guidance of teachers, and contains without doubt many valuable suggestions which to myself at any rate would have been of the greatest service had they been put into my hands when I began teaching twenty-five years ago. At the same time the suggestions seem to me to embody rather the experience of examiners familiar with certain types of error, than of teachers who have watched the gradual growth of ideas in their pupils' minds.

In order not to occupy more space than is absolutely necessary,

I will confine my remarks mainly to those suggestions in the report which appear to me of doubtful value, or in respect of which my own experience leads me to an opinion different from that expressed.

§ 2 suggests that the preliminary experimental work "should if possible aim at discovery—mere verification is less useful."

My own advice to a young teacher would be the contrary. I should say, beware of being led away by the enthusiasts of the heuristic school, into "playing at discovery" in such a subject as mechanics, especially at the beginning, when even the nomenclature is unfamiliar. The game appears to me in all subjects very liable to be misleading and to be unfavourable to sincerity of thought. Aim, on the other hand, in the laboratory work at careful verifications, extracting from the pupil himself a criticism of the accuracy of his own work and an enumeration and examination of probable causes of error. It is not at this earliest, but only at a later stage that you need be afraid of giving too much help. I am assuming that this early experimental work is so arranged as not to be a mere mechanical, drill-like repetition of a lecturer's operations, into which it seems liable to degenerate in the hands of some teachers, but leaves scope for individual initiative on the part of the pupil in arranging experimental conditions so as to enable him to test the point at issue.

Under the head of "General Remarks on Examples and Methods," we find in the report—

§ 7. "Prominence should be given to geometrical methods."

If the words "*as well as analytical*" were added, and if this recommendation only meant, that even in comparatively simple cases where geometrical methods are not the most accurate or easiest, they may yet with advantage be employed, side by side with analytical methods, for the sake of those pupils in a class who find the geometrical methods easier to follow, and who will by this means be led to a better understanding of the analytical process—if this were all, I should raise no objection; but the study of later recommendations in the report makes me think that the teacher is here invited to employ the processes of graphical statics at, in my opinion, far too early a stage. The fact that many problems which demand practical solution are too laborious to be dealt with analytically, but can readily be solved graphically,

is no justification for the use of graphical methods where they obscure the real issue, as they do in a course whose first object is not the solution of complicated problems, but the elucidation of first principles.

Under the specific head of statics are ten suggestions which do not to my mind convey quite the best way of teaching the subject. I agree with the first (No. 11), that "as the basis of the subject the parallelogram of forces should be assumed as an experimental result." But instead of recommending the teacher to pass at once to problems on equilibrium of bodies acted on by three concurrent forces, to be solved graphically (§ 12), followed by similar problems to be solved by calculation (§ 13), I should urge that after a very ew such illustrations the teacher should pass on without delay to the principle of moments, based, as suggested in § 14 *a*, on experiment. The reason for this course is that the choice of suitable examples, and especially of examples of a practical kind, such as are naturally attractive to a boy, is thereby enormously extended.

The pupil becomes accustomed at the earliest stage to deal with a finite body, which he *must* represent in his diagrams and on which the forces he deals with will be represented as acting at different places, whereas from examples which have to be artificially limited, so that they may be confined to concurrent forces, the body acted on is liable to vanish altogether from the mental vision of the pupil and there is a loss of the sense of reality about the work. If this reality in the examples is attended to, the pupil soon gains a sense of power which of all stimuli is the best.

But there is also another advantage gained. The range of examples being widened, it is easy to avoid the repetition of examples too closely resembling each other, and requiring too little thought for their solution. It is more important that the series of examples worked should seem different than that they should really be different. If each seems different, the student is pulled up at the outset and obliged to think of his underlying principles and to be careful in his statement of their application. When the new example is obviously just like the last, he is too often inclined to abbreviate, to hurry, and to cease to think.

It will be observed that, by adopting this order of treatment, the first use to be made of the parallelogram of forces is in

the deduction of the method of obtaining resolutes in any mutually perpendicular directions, which resolutes can be dealt with independently.

The recognition of this independence of mutually perpendicular resolutes is not so instinctive and immediate with beginners as not to require careful cultivation. In most practical cases the application to a body of a force in some given direction evokes constraining reactions in other bodies not previously active and results in motion in a different direction.

It is necessary that many such cases shall be carefully examined and explained before a boy can be expected to feel that his experience really bears out the principle, for the appeal must be made, not to the geometrical perception that the line *representing* the force has no resolute perpendicular to itself, but to the sensory perception that the *force itself* has no such resolute.

As to the suggestion (14 β) that parallel forces may be dealt with by the use of the funicular polygon, I have said enough to make clear my reasons for insisting that graphical statics ought to be postponed till first principles are firmly grasped. Unless this is done, graphical statics becomes a mere collection of recipes for finding a solution, which can be, and too often are, blindly followed, though quite imperfectly understood. With this proviso as to the order of instruction, I agree with the recommendation of § 15 that both graphical and analytical methods should be taught and numerical applications given; but I would leave for the technical schools and purely professional courses any extended drill in the application of graphical statics.

The remaining recommendations of the report under the head of statics do not seem to call for any special remark, but before passing to "Kinetics (commonly called Dynamics)," which is the next heading of the report, I should like to say a word as to the order of teaching on which the report contains two paragraphs, viz.:—

No. 37. "A short course of easy numerical trigonometry should precede the theoretical study of mechanics"; and

No. 38. "The Committee does not wish to recommend that statics should precede kinetics, or *vice versâ*."

As regards the first of these, I have no doubt whatever either of

its advisability or of its feasibility, and here I see that I differ (as in many points) from Professor Perry.*

On the other hand, as regards the second point I am in cordial agreement with him in considering that the study of kinetics, *i.e.*, of the operation of "unbalanced" forces, should *follow* that of statics. I have given the reversed order a careful trial, and should not dream of repeating the experiment, at any rate with pupils to whom the possession of a good working knowledge of the subjects was of real importance.

Those who hold the opinion that, in an elementary course, dynamics may with advantage be taught before statics, are, I believe, generally influenced by two considerations—(i) the desire to make use of the displacement of a given body that results from the action of a given force, or a given time as a measure of the force, and thence to deduce the theorem of the parallelogram of forces from the parallelogram of displacements; and (ii) the desire to make use of dynamical illustrations in the early teaching of the differential calculus. The first reason perhaps had weight in the days when it was considered highly improper to use the parallelogram of forces before it had been formally "proved," but the fact was overlooked that the dynamical experiments to be relied on were less easy to perform than the experiment of a direct statical proof. As regards the second (though, to be sure, only kinematical and not kinetic examples are really required), I see no reason why any law of dynamics of physics may not be stated and made the basis of an algebraical discussion long before the evidence which justifies the law has been studied. The rise and fall of a stone would interest more boys than the rise and fall of stocks. A great many physical laws could be tersely stated and made the basis of questions in text-books of arithmetic and algebra, and in this way an interest in the phenomena concerned would be very usefully excited. The matter is one which deserves more attention than it has yet received.

Coming now to that part of the report which deals with the teaching of dynamics, I find something to criticise or question in most of the items.

In the first place I do not understand what is intended by such

* Prof. Worthington is under a misapprehension; I advocate not less, but much more, preliminary mathematics than the M.A. Committee.—J. P.

phraseology as that of § 28, "It should be permissible to treat," or of § 36, "There should be no objection to illustrating the idea of a rate so as to lead up to the elementary ideas of the calculus." Have we no "lehr-freiheit" in this country that such words should be used?

What, again, I ask myself, leads anyone to encourage his pupils to write down, *as a simple proportion*,

$$\frac{\text{force acting}}{\text{acceleration produced}} = \frac{\text{weight}}{g},$$

instead of

$$\frac{\text{acceleration}}{g} = \frac{\text{unbalanced force}}{\text{weight of body acted on}}.$$

Is a boy really to be encouraged to write down the ratios of quantities of different physical dimensions and to regard it as a simple proportion?

Such a recommendation again as that of § 31 reads more like an extract from some examiner's report on some particular school than a serious suggestion for general guidance. I prefer, therefore, rather than deal in detail with recommendations with which I have little sympathy and the meaning of which must be to me sometimes a matter of conjecture, to set down some points which I have learnt to regard as important. In the first place the teacher can always rely on the fascinating interest of his subject. Let him indicate, when he begins, the kind of problems which he can promise his pupils that they will be able to solve after a few weeks' study, and let him occasionally feed them with further promises as the work proceeds, and he is sure of having a class who want to learn.

Then let him begin with kinetics, considering first the effects of different known forces (measured statically) *on the same body*. This will lead at once to the need of defining velocity and acceleration, but don't let him leave his kinetics for kinematics yet. Then let him pass to the effects of the same force on different bodies. This reveals the new property of inertia or mass, a property necessarily involving the consideration of *time*. The proportionality of mass to weight can be convincingly demonstrated by the simplest of experiments—that of dropping simultaneously a light and a heavy body. But it is important to emphasise that the reason of this proportionality is absolutely

unknown, and its exactness questioned. If the value of the acceleration of a freely-falling body is now assumed (to be verified later), there is already abundant material for numerical examples involving forces, masses, velocities, and accelerations—but not yet displacements.

At every step accurate definition of every new term is necessary. It is one of the great difficulties of the teacher that such terms as mass, weight, velocity, moment, are already in common use in an untechnical sense, sometimes in more than one sense; it is therefore most important to insist on the greatest care in the verbal expression of the argument used in the solution of any example, and for this reason I regard the best equipment for a boy beginning dynamics to be a good previous training in language and in the accurate use of terms. It is not the numerical solution of an example that is so important at this stage, as the accurate writing down, *in words*, of the equation which leads to it, and the criticism of this wording I regard as the most important part of the teacher's work.

After such examples have been practised, the idea of a body moving with uniform acceleration will be sufficiently familiar to lead in a natural way to a short special study of the motion of a particle moving with uniformly accelerated velocity in a straight line.

This will enable the pupil to extend his range and solve examples in which he can now deal with the *displacements* concerned. I have found that much is gained by thus leading a class up to this elementary kinematics. It is also important to insist on a clear distinction between the purely kinematical part and the kinetic part of any problem.

It is to be recommended that this elementary kinematics shall form part of the regular mathematical course, which may well be made to include the unresisted motion of a projectile and uniformly accelerated rotation about a fixed axis. If the instruction in dynamics and in mathematics is in the hands of the same teacher so much the better; if not, co-operation is necessary, and will be found of the greatest use.

If I may dwell on a few details, I would mention that I have found it very useful to distinguish between pound force and pound mass by writing "lb." for the mass, and "pound" for the

force.* To use the symbol "*g*" simply as an ***abbreviation*** for "the acceleration of a freely falling body"; very frequently to speak of the "earth-pull" on a body instead of its weight; to coin such words as ton-als and ounc-als when necessary; and I have ventured to term a "slug" the mass to which a force of 1 pound gives an acceleration of 1 foot per second per second, which is conveniently called a velo per second or a celo.

These may be among the "fancy names" condemned in the report, but I should be sorry to be without them. They all conduce to clearness of expression and of thought while the pupil is learning.

So far as I have outlined, no difficulty attaching to the units of measurement need present itself, nor should be allowed to present itself; the methods of calculation used have been quite untechnical. Force has been regarded as proportional to the mass-acceleration it produces, the necessary constant factor being provided by an observation of the acceleration of a freely-falling body; the one new idea that has been presented being that of inertia or mass. It is only when we come to "momentum" and have to teach the pupil to think of force as $K\frac{d(mv)}{dt}$ instead of as $Km\left(\frac{dv}{dt}\right)$, and to point out the convenience of making K equal to unity that the difficulties belonging to conflicting systems of units need present themselves. These difficulties require, of course, to be patiently faced, but they are not difficulties of dynamic, but are chiefly linguistic, arising, *e.g.*, out of the fact that such terms as "lb." and "pound" have the same sound, that the word "gram" may mean either a force or a mass, and that for units of momentum and mass-acceleration we have no suitable names at all.

Especially in dealing with any numerical equation involving work and kinetic energy beginners are peculiarly liable to forget to express all the terms in the same units; some terms may be most easily written down in absolute, and others in gravitational units of work. I have found it useful to teach students to

* According to this convention we should say that a body "weighs a lb.": the word *weigh* meaning originally to move or lift (*cf.* "weigh anchor"): to "weigh a lb." means to lift a lb. when placed in the opposite pan of a balance.

write down at once the denomination of any such term at the side of or above it, but am reluctant to set down methods of avoiding mistakes, seeing that nothing is more profitable to a student than the making of mistakes, and unless the difficulties I have alluded to are fairly met and mastered, and security gained by repeated failures followed by renewed efforts and watchfulness on the part of the pupil, they will rise up against him later on in his work, to his confusion and discouragement. This training in the careful handling of dynamical expressions, and in the clear thinking required, has in my opinion a very high educational value.

It is unfortunate that in the earlier parts of dynamics good experimental illustrations are not easy to devise. Thus I have not yet succeeded in finding any satisfactory way of subjecting a mass to the successive action of different known forces and determining the acceleration in each case.

For a first determination of "g" in the presence of a class, I can recommend the use of the trace made by a style, attached to a vibrating tuning fork of known pitch, on the smoked face of a massive glass plate let fall in front of the fork. Later on, when rotational motion is reached, experiments are easier to find. A good many will be found described in my *Dynamics of Rotation*.

I should like now to answer Professor Perry's inquiry as to the age at which a boy may be expected to understand that "force is rate of change of momentum," by saying that the age will be the same as that at which he may unfortunately be misled into thinking that the word "is" has the same meaning as the words "is measured by," or "is proportional to," and that Newton's Second Law of Motion is mere tautology.

The distinction is so important, if we are to escape from a vicious circle (in which even M. Poincaré seems willing to wander), that I hope I may be excused from calling attention to it.

Otherwise I would reply to the real drift of Professor Perry's question by saying that, so far as my experience goes, the habit of instinctively correlating force with rate of change of momentum is only to be acquired by long and varied practice in many branches of mechanics.

In conclusion, I desire to record a protest against a tendency among those who, I think, can have little understanding of a

teacher's work and little sympathy with it to decry the use of what they term "academical problems" in mechanics, a term which is not indeed very definite, but which I understand to refer to problems from which by an act of imagination troublesome conditions, which would certainly in practice disturb the result, are supposed to have been removed. It is too often forgotten by such objectors that hardly any practical problem in applied mechanics is capable of exact solution, and that the only way of obtaining an approximate solution is to make such assumptions and restrictions as will reduce the matter to the solution of a despised academical problem. The greater the variety and range of a student's experience in such problems, the better is he armed for attacking more complicated practical cases.

Professor PERRY. (Reply):

At Johannesburg I was urged by several speakers to explain my objection to the poundal and my acceptance of "centrifugal force." In *Nature*, November 19, 1896, in reviewing a book, I spoke of engineers' units, and during the next four months the editor of *Nature* received and published letters on the subject from Lodge, Fitzgerald, Greenhill, myself, and many other writers. I hope that these letters may be consulted.

My students work with two sets of units, with the C.G.S. and with engineers' units. They are one as absolute as the other; they can be used at once in any formula, such as $F = \frac{d}{dt}(mv)$ or $E = \frac{1}{2} mv^2$ or $E = \frac{1}{2} Ia^2$. In the one case answers are in dynes and ergs, in the other the answers are in pounds and foot pounds.

In all English-speaking countries men who apply scientific principles use the pound as the unit of force and the foot pound as the unit of work. Other units employed for convenience, such as the ton and the horse-power hour, are founded on the two fundamental ones. I know of no engineer or other practical man who does not habitually employ the pound as the unit of force in thought and when expressing his thoughts, although I may say that a few—a very few—who teach students, work exercises in poundals and convert their answers into pounds at the end. Let my critics bear these facts well in mind, as they are of fundamental

importance. Next, may I say that I have charge of the examinations in the seven engineering science subjects of the Board of Education; I have something, therefore, to do with the instruction of perhaps 150,000 students. Candidates are allowed to work exercises through poundals or foot poundals if they please, but it is quite usual to find, I may say that in 50 per cent. of the cases we find, that in converting their answers they multiply by 32·2 when they ought to divide, or they divide when they ought to multiply. I am afraid that I must ask my critics to take this also as a fact.

Whether boys are or are not going to be engineers, they know that they would excite ridicule if they spoke of poundals or foot poundals to ordinary practical people; the results of all their calculations must be reduced to pounds and foot pounds.

It may be said: Let schools and colleges stick to poundals, and the public must give way in time. This is a delusion. The engineer has already a contempt for college teaching because it is too remote from practical applications. Teachers who use poundals are doing the very greatest possible harm to scientific education just because they justify this contempt of the practical man.

But, say our critics, the poundal is fundamentally right and the pound is fundamentally wrong. Now, if this were so I would be the first to fight on the side of the poundal. I have never been afraid to fight the practitioner on matters of fundamental importance where I thought him wrong, but I believe that in this poundal matter the practitioner is right and his critics are altogether wrong.

In engineers' units the weight of a standard piece of stuff in London is called one pound, and this is the unit of force. If the piece of stuff were taken to some other place on our earth its weight would differ so little from its weight in London that it is only in a very few physical investigations that the difference is of importance.

The unit of inertia (inertia is now stupidly called mass and denoted by the letter *m*) is called a *slug*, being 32·2 times that of the standard piece of stuff, because it has been found by experiment that the unit force produces upon it an acceleration of one foot per second per second. Hence in engineers' calculations the numerical value of *m* in slugs = weight in pounds

in London ÷ 32·2.* These units with the foot and second may be employed in any dynamical formula, and there is no curious factor needed. Professor Greenhill is greatly with me, but he objects to the use of m or the idea of mass (inertia) in all dynamical formulæ. I cannot agree with him in this, and I think he is alone. It is most important to introduce kinetic ideas in the form:

Acceleration $a \propto$ force F. Now weight w is a force producing the acceleration g; therefore

$$\frac{F}{w} = \frac{a}{g}.$$

But surely when a boy has thought about this we may go further and say

$$F = \frac{w}{g} a,$$

and call $\frac{w}{g}$ the inertia (or mass) of the body, denoting it by the letter m, so that always $F = ma$.

But I have no dispute with Professor Greenhill. I say that the engineers' units as above defined are just as absolute as the C.G.S. or any other so-called absolute units, and a man who took a spring balance graduated in pounds and a foot-rule and a good chronometer to the planet Mars would have no difficulty in using engineers' units there. If he cannot get rid of the chemists' prejudice, and loves to speak of that metaphysical thing, "quantity of matter," let him leave the spring balance behind and take a standard *slug* instead. If my critics are honest, they must admit that the above statements are correct. Well, then, why do they regard the poundal as a thing which is fundamentally important? Because, as the gramme is the unit of mass in the C.G.S. system, the pound ought to be the unit of mass in the other system. Is not this mere analogy the reason? When I was a schoolboy I talked of the inertia of a body and moment of inertia. The chemist has now introduced the notion of "quantity of matter" or mass, and the idea that this is a simple business is the basis of much cant and all our trouble.

Many teachers cannot comprehend how anybody can avoid

* To correct Mr. Macaulay, I do not take g lb. as the unit of mass. I take 32·2 lb., assuming that g in London is 32·2 feet per second per second. The more correct value is 32·182.

using the pound of stuff as a fundamental unit of mass or quantity of matter. The inventor of the poundal, the late Professor James Thomson, thought of *inertia*, modern people think they follow him when they speak of mass or quantity of matter. As I said in *Nature*, December 24th, 1896 (the same ideas are given with greater authority by Professor Fitzgerald, February 25th, 1897), "A pound of force, a pound of "stuff, the inertia of a pound; here are three very different things "all with the same name. When the chemist tells us that there "is the same quantity of matter after as before chemical combina-"tion, what does he mean? He means that the *weight* of it is "the same: the force of attraction of the earth. A certain "amount of oxygen is equivalent in a certain property (its weight) "to a quantity of hydrogen, and he says he will call the quantities "equal. Certain quite different amounts of them are *equivalent* "in another property; he has exactly the same reason for calling "these other amounts equal. A ton of iron is equivalent in a "certain property to two ounces of gold. Why not call these "amounts equal?

"A pound of gold is no more the same as a pound of iron, "because their weights and inertias are the same, than two chairs "are the same as one table because they may be equal in value. "I hope that Professor Fitzgerald may be induced to say some-"thing on this head, the 'hugger-mugger' of confounding quan-"tity of matter with inertia, for I think with him that this is what "produces far more confusion in the minds of students than the "use of many different units for things of the same kind. The "practical engineer has uncommon good sense—he hates the "poundal; and I think that Professor Fitzgerald is right when he "says that it is not merely because it is a new unit, but because "it is founded on 'hugger-mugger.'"

The *Nature* letters are too long to quote, and they are easily accessible. I must confess that the contempt of my opponents was very galling to me. Not one of our critics seemed to me to have taken the trouble to even try to understand what Greenhill and Fitzgerald and I, and others who were on my side, wrote. Their attitude towards us was that of Kingsley to the root-eating Irish, my own attitude towards the teller of a true ghost story, or towards a flattener of the earth, or towards the inventor of a perpetual motion.

Now, we understood the academic position perfectly well, for we had occupied that position ourselves, and we had all had experience in teaching boys. What I complain of is that our opponents were altogether ignorant of one side of the case, yet they were acting, not as mere advocates, but as judges.

I say that our critics have a certain scholastic point of view, and they cannot take any other. *Mass*, quantity of stuff, is to them the fundamental idea, and they cannot conceive of its being a mere metaphysical abstraction. Although all their measurements are through *force*, although a boy has been brought up to think about forces all his life, they insist on teaching him, not through his own notions, not through his past experience, but through abstract notions which cannot become familiar to students until long after they have begun their study of kinetics. The mind of the average boy is very different from that of the usual teacher, and the teacher calls the boy stupid because he cannot take in the idea of inertia through mass. It is exactly the same mistake that was made in teaching geometry, and teachers of mechanics are now as unable to see the average boy's point of view as teachers of Euclid used to be. The average boy used to learn no mathematics, and was labelled a dunce; it is now found that he is no dunce, and he does learn mathematics. At present the average boy learns nothing of mechanics—he is such a dunce! But if his teachers would only try to teach him through force, through weight, making mere weight the fundamental notion at first, they would find every boy capable of learning mechanics, and the boys usually called clever would benefit too.

I wonder what answer Mr. Macaulay gave to his inquisitive pupil. Surely he had to say that the moment of inertia of his wheel depended upon mass (inertia) and dimensions, and mass is always deduced from gravitational weight, using the doctrine that the weights of equal masses are equal.

Suppose a man to see a wheel upon the planet Mars, can he state its mass in pounds unless he has a pound weight with him? Well, he can state its mass in slugs if he has a specimen slug with him, or if he has a spring balance with him graduated in pounds he can arrange to measure its inertia in slugs. Critics who take absolutely no trouble to study what they criticise say, "The weight of a pound would be quite different on the planet Mars; the mass of a pound is everywhere the same, therefore engineers' units are

absurd!" The weight of one pound at London, the place where the standard piece of platinum is kept, this is our exact definition; and if the standard piece of platinum might be carried to Mars, why might not our slug be carried, or our graduated spring balance?

I have been told that we are introducing "a new and unpractical inertia unit, based upon the intensity of gravity near London." When men of great scientific reputation criticise in these ways it is not easy to postulate the value of physical science study in general education. Is the intensity of gravity at London then a varying thing? On what are any of our units based? Our yard and pound are based upon destructible things kept in London. The gramme and centimetre are based upon destructible things kept in Paris. The second is based upon astronomical measurement of something which is certainly altering. And my friend calls the engineers' units "new and unpractical," although they are as old as the oldest dynamical calculations and are employed by all practical men. It is the poundal system which is new, and it is never employed by practical men (see *Postscript*).

Now as to *centrifugal force*, why forbid its use? I cannot imagine a method of teaching so bad that the use of this term occasionally will hurt the pupil. A body A constrained by a string B to move in a circular path, is acted upon by B with centripetal force, but surely A acts upon B with a centrifugal force. The two forces constitute the stress in the string. The fact is, the use of this term centrifugal force is natural to a boy or man, it expresses his experience; and when you forbid a boy to use it you may be said to close up all access to his mind: he gives it up, he gets the notion that this is another subject which he cannot possibly understand. The centripetal force exerted by some medium, let us call it the ether B, upon the earth A is surely equal and opposite to the centrifugal force exerted by the earth A upon the ether B. Of course, if a teacher has been brought up on the idea of action at a distance, and if his study of mechanics has been altogether through astronomy, he is likely to object to the term centrifugal force. Again, in the highest applications of mathematics it is often most important to convert a kinetic into a statical problem, and it is then natural and usual to say "the centrifugal force acting on the element of stuff . . . ." I can quote such expressions from the most orthodox Cambridge

text-books. Dr. Chree or Mr. Whittaker uses the very language which Mr. Godfrey thinks ridiculous. I need hardly say that I myself do so also, and I cannot believe that any pupil has ever been misled by such language. And I affirm that almost every kinetic problem concerning bodies moving in curved paths is converted into a statical problem by everybody who wishes to solve it, and consequently it is centrifugal and not centripetal force that has to be more often talked about.

Critics who dwell upon the fact that at the present time boys of fourteen or sixteen have not the mathematical power or knowledge of mechanics which I expect them to have seem not to have read the first four lines of my address. Surely we need not argue as if the present neglect of science in school education is to continue! It is quite possible for the average boy to have a good working power in English subjects, in mathematics, and in natural science; these are the only *essential* things in a liberal education, in my opinion. If I am right, the standard boy of Mr. Siddons will soon disappear. Mr. Siddons says "the calculus is a powerful tool but a dangerous one in untrained hands." No doubt; and so is a knife. Whether a boy is learning to use it early in life or late, there is a time when he makes mistakes, and through these mistakes he becomes trained. I say he may be trained quite early. Mr. Siddons thinks that I introduce greater complexity than the M.A. Committee; my opinion is just the reverse. Mr. Eggar argues as if I wanted a "purely mathematical treatment" of mechanics, but he cannot surely mean this. He says, "You cannot do without kinetics at the start, because in your static experiments things move." This kind of argument may be extended. Do we cause a boy to make a formal study of logic because we cannot do without logic at the start? Or of acoustics because the baby makes a sound at the start? I am glad he is willing to give up the poundal if the engineers won't have it. Well, that is a certain thing; the engineers will have nothing to do with it!

I agree with Mr. Jackson, and he seems to agree with me. I ought not to have omitted what he and Professor Minchin mention, the interest which a class of boys will take in the anecdotes of the history of a science. I quite agree with him in what he says about examinations. Professor Bryan objects to so many boys being taught mechanics—so many are incapable. I say that if my method is followed surely it must be good for all boys to

learn something of measurement and how to use their hands. One boy in a hundred may be unfit, as one in a hundred is unfit to be taught to speak or write. What is the use of talking of overloading the competent teacher with a lot of dull boys? The man who cannot teach the average boy ought to be called an incompetent teacher who dishonestly takes money from parents for doing work which is never done.

In answer to Dr. Lees, I would say that for elementary students there is more educational value in cheap apparatus than in what is expensive. Twenty-five years ago I bought a large, rusty, old, unbalanced pulley at a marine store, and since that time it has given excellent experience in the measurement of its axle friction to thousands of students. The apparatus designed by me in those days has been copied and is now being used in many parts of the world. I notice that in such things as the pulley, friction has been diminished as much as possible, so that to discover the relation between friction and load requires most careful manipulation. The cheapest form of Attwood's machine has greater educational value than the most expensive.

In the opinion of nearly all my critics the average boy cannot possibly understand that *Force* is equal to or proportional to (vector) rate of change of momentum, and this boy who must for ever remain ignorant of the fundamental law of mechanics is said to be well acquainted with *centripetal force!* The thing is impossible! It is my opinion that from the very beginning we ought to aim at a thorough comprehension of this greatest and simplest of all laws. Without the help of this "open sesame" the student may cry "open barley" in vain when confronted with a new problem. The simple idea is easy to give experimentally, once for all. I complain that teachers never give it because they themselves have no clearness of vision about mechanical things.

(*Postscript*).—In the discussion upon Mr. Ashford's paper it was announced authoritatively that in future in our Navy the officers will calculate in *poundals*, An artificer will therefore be told to keep the boiler pressure at so many poundals per square inch! Or may it be that there will be an interpreter provided who will convert this Choctaw into English? The academic kid glove officer talking a language other than English would not have delighted Francis Drake. This is not a small matter, it is a great mistake which may have far-reaching consequences.

THE TEACHING OF MECHANICS BY EXPERIMENT.*

By C. E. ASHFORD, M.A.
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IN spite of the developments in the methods of teaching the physical sciences, mechanics is only just beginning to be affected. So far as public schools are concerned, this is probably due to the fact that it was for generations in the hands of mathematical masters, who naturally treated it as a deductive, not an inductive or experimental, science. Given the principle of the parallelogram of forces and Newton's laws of motion, they trusted confidently in their own mathematical powers for developing the science on these foundations. It would have implied something like a slur on the power and accuracy of mathematics to appeal to experiment, and this feeling was so strong that no stress was formerly laid on the experimental basis even of the laws and principles on which the science was built up.

The result was an elegant and rigorous treatment of the subject, which formed an excellent training so far as it went, but which lacked interest and left unattempted what is perhaps the most important result of teaching a science properly.

When experimental science was thrust into the school curriculum through the pressure of outside opinion, the introduction of the masters who taught it did not modify this one-sided treatment, for it happened that these men were for the most part chemists. For many years science and chemistry were synonymous in schools; the science master was not unwilling to be relieved of the responsibility for teaching mechanics, and the

* Extracts from a lecture delivered at the British Association meeting at York.

subject remained a part of the mathematical course, usually learnt only by the better mathematicians.

With the advent of the physics master came strong pressure to develop the experimental side of mechanics; this has been done in many schools, but hitherto it has been subservient to the mathematical treatment, designed to illustrate and perhaps to verify the results deduced by other processes. No attempt is usually made to attack the subject inductively; the apparatus is merely the text-book diagrams in three instead of two dimensions. In addition to this, the experimental treatment is practically confined to statics.

If we turn to the other group of schools that is engaged in the teaching of mechanics, the technical schools, we see exactly opposite methods. There mechanics is treated frankly as an experimental science; the pupils learn much that no public-school boy derives from his work in class-room or mechanical laboratory, but often miss the special training given by the public-school course of mechanics.

Is it a Utopian ideal that these two lines should converge, and in their junction should admit of the development of a method of teaching mechanics which will provide for the three desiderata, a knowledge of the facts and principles of the science, a strengthening of the reasoning power, and a training in the proper way to study natural phenomena in order to fathom the underlying causes? I believe that this can and will be done.

The reforms in geometrical teaching, proceeding from masters in the public-schools, show that mathematicians have realised the great value of a preliminary experimental training even for the most crystallised of the deductive sciences. The same process must be adopted in the early training in mechanics, but the difficulty here is that the methods must be different. What is needed in this early course is the influence of the engineer with his experience of actual practice, not that of the schoolmaster with his traditions of text-book toys and models. It is difficult to say how much the schoolmaster and the schoolboy owe to Prof. Perry in this matter. His insistence on the necessity of starting by an investigation of actual machines has already begun to influence school teaching, and the course he advocates is

exactly what is needed to form the foundation on which the mathematical master may build, as well as to supplement the course he gives.

I should like to describe here an interesting attempt to combine some of the features of a technical and a public-school.

The Admiralty has established colleges at Osborne and Dartmouth for training naval officers. Now, the naval officer has to be a many-sided man, and not the least important side is that of an engineer. It is of vital importance that his engineering should not be merely book-learning; he is a man of action, and a large employer of skilled labour, in a ship that is "a box of engines." Hence he must receive a sound practical as well as theoretical training in science and in mechanical and electrical engineering. The problem of giving it to him is complicated by the fact that due attention must be paid to many other sides. We may neglect these for the moment, but they must not be left out of account in arranging for any one branch.

Granted that in the ordinary process of training engineers both a college course and a workshop course are essential, much discussion has taken place as to their order of sequence. The Spartan *régime* of evening classes following laborious days at the bench is now recommended for the Admirable Crichtons, and this with important modifications is adopted for all naval officers *in statu pupillari.*

It is not the custom for great engineering firms to arrange their establishments solely with a view to the most rapid progress of their premium pupils. They own workshops, not educational establishments. In a workshop which *is* an educational establishment very many hours may be saved without detriment to the shop training. These hours may be employed in carrying on concurrently the scientific as well as the general education, thus providing that excellent recreation, a change of occupation, and making it possible to ensure that the chain of subjects taught, from the most technical to the most humane, should be a true chain, with each subject linking into and depending on the next.

The cadets spend eight or nine hours a week in the workshops during the whole of their four years at Osborne and Dartmouth. Considering for the moment only the effect this has on the teach-

ing of mechanics, it will be seen that for any mechanical principle which they learn in the lecture-room the cadets see many practical applications the next time they go into the workshop. Again, they come to the lecture-rooms with a store of practical knowledge of machinery and materials on which the lecturer can base his teaching. There is available, too, a storehouse of knowledge on which the mathematician can draw for his examples, preferable in every way to the problems on finance, clocks, and adulteration of grocery which used to swell the text-books of arithmetic and algebra.

It is evident that these conditions are very favourable for carrying out such a plan as I have suggested, of building up a logical and mathematical superstructure of mechanical knowledge on the foundations of an acquaintance with real machines and processes. It would not be fitting for anyone connected with the building process to attempt to estimate its success, but I may draw attention to some of the points which have impressed themselves with regard to the laboratory and lecture-room teaching of mechanics, possible under more ordinary conditions.

I may say at once that I have no sympathy with the fashion for teaching boys kinetics as an introduction to statics. It is a highly convenient sequence in which to develop the subject for anyone who already possesses a general knowledge of the whole, and may commend itself to the metaphysician, but it does not do so to the ordinary boy. The latter finds it easier to think about and to experiment with bodies at rest or moving with a small uniform velocity. He starts with a good working idea of a force; it is a long time before he is anything but hazy about an acceleration. To his mind a force is chiefly occupied in preventing motion or in keeping up fairly uniform motion against a resistance; in his experience uniform acceleration is rare. I venture to suggest that it is comparatively rare in the experience of most of us, and that we have evidence of this in the continual harping of text-books on bodies falling freely and trains getting up speed.

I believe that most boys in their heart of hearts always believe that a body loses its weight when falling freely; how, then, does he grasp the idea of a force producing its acceleration? And if he inquires how sundry great principles really are established, he

is referred to a planet or a pendulum, of which the acceleration is none too constant.

Assuming that we begin with statics, we must use the boy's own muscular sense and experience to give him the notion of force. But a spring balance must be called in to help him to make quantitative experiments. He will learn little at first from a lecturer, but will need a good deal of guidance in the interpretation of his experiments. He can easily rediscover and verify to his own satisfaction the parallelogram and triangle of forces. It appears to me to be a perfectly sound compromise to let him rest content with such a verification, warning him that it is not a proof, but that he will be able to prove it when he can apply the laws of motion; and when later on he has grasped these and perhaps understood why we trust them, we can bring him back and give him a rigorous proof. This is only a first course, intended to give the learner vivid and accurate ideas, and to give him a chance to use his drawing instruments as well as his imagination and powers of analysis. I do not suggest that, after the foundations have been well and truly laid, the superstructure should not be erected according to the traditional methods; I believe that in the modernising tendency of the technical school there lies a serious danger of forgetting this, and of making no provision for a proper study of more advanced mechanics. The same danger has been guarded against in the revolution that has taken place in the teaching of geometry; it is to be hoped that the influence of the public-school teacher will effect it in mechanics.

The average boy's knowledge of the mechanics of everyday life will make lectures on levers and the moment of a force easy and interesting. The extent to which he can deal with energy depends on the amount of science he knows already, but he will have no difficulty in comprehending the idea of work and its measurement, when illustrated with actual machines. He can early experiment with such machines as a screw-jack, block and tackle, differential pulley-block, or halley-jack. Working with these will make him think, and he will do it cheerfully because he is dealing with actualities, not toys and models, and because he can reduce his experiments to a form which when plotted on paper will tell him things worth knowing. For a description of how

such work can be done I would refer to the excellent *Practical Mechanics* of Mr. S. H. Wells.

Now, the ordinary school course omits this most important and interesting part, which probably teaches a boy more real mechanics than all the rest. It certainly shows a boy how he must set about studying the complicated mechanisms by which he will be surrounded in later life. Unfortunately, school laboratories are stocked with elegant models of these machines. The scientific apparatus maker sends his lists to schools, the maker of tools does not, and that, I believe, is the true cause of the fanciful nature of the mechanical apparatus still to be found there. Curiously enough, the model not infrequently is more costly than the genuine article. To take an example, the list price of a wooden model screw by a scientific instrument maker is 15*s*., that of a brass working model of a screw-jack is £2, and the list price of a maker of the commercial article is 6*s*. for a 1½ ton jack. But I believe that most schoolmasters would think it as unreasonable to order some heavy weights and half a dozen screw-jacks as to order half a dozen locomotives for laboratory use. It is hard to break away from the glass-cased balance, and the feeling that a hundredweight is out of place in a well-conducted laboratory.

But perhaps the most marked feature in the teaching of mechanics is the lack of experimental illustrations of kinetics. The whole of the earlier parts of kinematics and kinetics usually consists of appeals to a boy's imagination and mathematical reasoning powers, eked out by references to some few phenomena of daily life, with perhaps an exhibition of the Atwood's machine and the pendulum.

Teachers know only too well that, from the point of view both of theory and practice, Atwood's machine is not well adapted for boys' use.

Recently Mr. W. C. Fletcher put into practical shape some experiments which any boy could perform and understand. He employed a long trolley carrying a strip of paper which was marked in transit by an inked brush on one end of a spring, rigidly clamped at the other end. This spring vibrated about eight times a second, and gave an easily measurable trace on the slowly moving trolley.

Of course, there was nothing really novel in the idea, but the novelty lay in its adaptation to the needs of a boy in a laboratory. This constitutes its great merit, and however roughly it be made (if a fairly true plane is provided for it to run on), a boy can get with it results correct to 2 per cent. or 3 per cent. after he has made the adjustments necessary to eliminate the errors which he can see and guard against. It does away almost entirely with the difficulty of timing the motion of a body. Consequently, since the same apparatus suffices for a number of experiments, and is not costly, no one should now plead poverty for leaving his boys without the means of illuminating by their own experiments a very difficult subject.

I propose to give a list of some few of the uses to which Mr. Fletcher's trolley can be put. I feel some shame in doing so, since it suggests that I am responsible for them. As a matter of fact, all that is new is entirely the work of my colleagues at Dartmouth; I have only been an interested looker-on. One remark applies to all: they are designed for use by the boys themselves, and have been extensively so used; they are not mere lecture-table instruments.

The first slide shows one of the ordinary trolleys on its plane. With this a boy can produce and measure uniform velocity and acceleration. Every experiment with a trolley provides him with a space-time curve ready made, and he can plot his velocity-time and acceleration-time curves from it. He can prove easily and with considerable accuracy that with a constant force the acceleration varies inversely as the mass, and that with a constant mass it varies as the force. With a suitable trigger, holding a pin on the trolley, and actuated by the spring in the middle of its first swing, he can get the motion from rest, thus simplifying his calculations and improving his curves. The friction of the trolley he can eliminate either by tilting the plane to the necessary angle, or by attaching a weight that will just maintain uniform motion, as shown by eye or, still better, by a preliminary tracing with the spring. The friction of the pulley over which the thread passes cannot be compensated, so that it is necessary to use a good pulley.

The objection to all such experiments on forces and accelera-

tions still exists, that a boy may feel that the accelerating force may be lessened by the fact that the weight is falling. Of course, he adds the mass of the weight to that of the trolley, so that it is not a real error; he may be able to grasp this, and in any case the velocity acquired is so small that it is not likely to present itself as a difficulty to the ordinary boy.

If a long spiral spring be fixed to one end of the plane, and the movement of the other over a scale of inches be observed as the spring is stretched with different weights, a simple indicator diagram in the shape of a triangle is obtained, from which can be found the potential energy stored in the spring for any extension. If the free end of this spring be then attached by a thread to a loaded trolley which is pulled back, on releasing the trolley and finding its final velocity, and hence its kinetic energy, the equivalence of this to the potential energy of the system, as previously determined, may be shown. If two trolleys are set side by side on a gently sloping table, and carry a board set at right angles to them, along which a third trolley can be dragged by a falling weight, it is easy to see and measure the composition of two accelerations. The third trolley may carry a brush which marks a straight line on the table.

With two trolleys on a long plane, each with its vibrating spring, one moving and the other initially at rest, he can verify the law of conservation of momentum, both for trolleys that stick together and for a pair that rebound. If a pistol be fitted on one and a target on the other trolley, on firing the pistol by a simple electromagnetic attachment, and measuring the momentum produced in each trolley, the same law can be shown to be true for the recoil of a gun. It is not necessary to say that this is a popular experiment with boys; it is also easy to make it a safe one.

With a plank pivoted about a vertical axis and carrying a strip of paper which can be marked by a vibrating spring, and an ivory stud which can be struck by a similar stud on the end of an ordinary trolley, a boy can measure the loss of momentum of this trolley on hitting the plank. He thus can find the moment of the blow on the plank, and can compare it with the moment of momentum acquired by the plank, thus finding the moment of inertia of the plank about its axis.

If he attaches a thread to each end of the trolley on a long plane, passing these threads over pulleys on the ends of the plane, and hanging to each thread a heavy chain the other end of which is hung from the plane so that the chain forms a U, the trolley will move with simple harmonic motion if disturbed. In practice friction must be eliminated by tilting the plane to the required amount, and the motion observed while the trolley makes one excursion down the plane. In this way he can obtain a velocity-time curve remarkably near to a sine curve, or an acceleration-space curve very nearly straight.

It is usually found that boys are not sorry when the question of the rotation of bodies about fixed axes is dealt with somewhat lightly. But it is not a matter that deserves such treatment, and if the parallelism of the equations for angular and linear motion is fully brought out, and if the boys make plenty of experiments with a "circular trolley" such as is shown, the difficulties seem for the most part to disappear. Experiments with this trolley have great value in clearing up a boy's notions of angular velocity, &c.; he can either take his vibration curves on tracing paper over a scale of radians and its subdivisions already marked on the trolley, or he can measure the wave-lengths in radians by taking the centre line of his curve at some definite distance from the axis, say six inches, and measuring the wave-lengths with a flexible inch measure. This effectually impresses on him the meaning of a radian.

There is a difficulty in this case that does not occur with a trolley used for linear motion, in that the mass of the falling weight cannot be added to that of the trolley. This can be avoided by using as constraining force the pull of a spiral spring, and using a heavy flywheel instead of a trolley. The interchange of kinetic and potential energy is observed as previously described, and gives a direct measure of the kinetic energy acquired by the wheel. Here we can eliminate the effects of friction in the bearings by attaching a suitable weight to a thread round the axle; it is now a question of the kinetic energy of this weight, which the boy can calculate, and disregard, if small enough, or add to that of the flywheel if necessary.

Exigencies of space curtail the list of experiments that have

been worked out to illustrate the principles of kinetics by the staff at Dartmouth; perhaps enough has been mentioned to show that as many are possible and simple in kinetics as in statics. It is in the hope that this use of trolleys may receive the development which it deserves that I have brought up the question. So far as I know, the only published references to it are in Mr. C. W. Fletcher's original paper in THE SCHOOL WORLD for May, 1904, and in Mr. W. D. Eggar's *Mechanics* (E. Arnold).

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